$f(1.5) = 3(1.5)^2 - 9(1.5) + 4$ M1

Minimum value is $= -2.75$ A1ft (G2)

(M1 is for substituting their value into $f(x)$. An answer without working is awarded full marks, G2.)

- **Follow through marks**

  Follow through marks (ft) are awarded in situations where an incorrect answer from the previous part is carried through to the subsequent part.

  If no additional errors are made, the student receives full marks for the subsequent part, provided working is shown.

  Possible follow through marks are indicated on the mark scheme.

- **Some Examination Reminders**
  - Unless told otherwise in the question, all final answers should be given exactly, or else correct to 3 significant figures.
  - For questions involving money, final answers can be given correct to 3 significant figures, to the nearest whole value, or to 2 decimal places.
  - Your calculator display can be set for 3 significant figures or for various decimal places. It can also be set for scientific notation (standard form).
  - Ensure your calculator is set to 'deg' for degrees. This will need to be done if your calculator is reset. For TI calculators, 'diagnostic' also needs to be set to 'on' if the calculator is reset.
  - Ensure your calculator is set to 'trig' for graphs of periodic functions (sine and cosine).
  - Use sensible view windows for sketches and general problem solving with graphs. Many students begin with "std" with a domain and range of -10 to $+10$. Negative values for the domain are often not appropriate for problems simulating real situations.

SOLUTIONS TO SPECIMEN EXAMINATION A - PAPER 1

1 **a** $5, -5, \sqrt{16}$ A2 **C2**

b $\frac{1}{3}, 5, -5, \sqrt{16}, 0.\overline{6}$ A2 **C2**

c $5, \sqrt{16}$ A2 **C2**

[6 marks]

2 **a** $f : x \mapsto 2x^2 + x - 3$

$\therefore \quad f(x) = 2x^2 + x - 3$

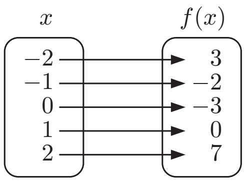
$f(-2) = 2(-2)^2 + (-2) - 3 = 8 - 2 - 3 = 3$

$f(-1) = 2(-1)^2 + (-1) - 3 = 2 - 1 - 3 = -2$

$f(0) = -3$

$f(1) = 2(1)^2 + 1 - 3 = 2 + 1 - 3 = 0$

$f(2) = 2(2)^2 + 2 - 3 = 8 + 2 - 3 = 7$



A2 **C2**

b Domain $= \{x \mid -2 \leqslant x \leqslant 2, x \in \mathbb{Z}\}$ A1ft A1ft **C2**

c Range $= \{-3, -2, 0, 3, 7\}$ A1ft A1ft **C2**

[6 marks]

3 **a** Surface $= 9.85 \times 5.90$ M1

$= 58.115 \text{ m}^2$

$\approx 58.1 \text{ m}^2$ A1 **C2**

b Rounded measurements are 10 m by 6 m.

Surface area $= 10 \times 6$

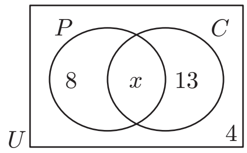
$= 60 \text{ m}^2$ A1

Percentage error $= \dfrac{60 - 58.115}{58.115} \times 100\%$ M1 A1ft

$\approx 3.24\%$ A1ft **C4**

[6 marks]

4 **a**



Total number of students $= 8 + 13 + 4 + x$

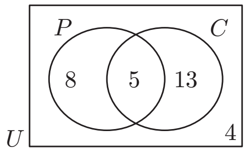
$\therefore \quad 30 = 25 + x$

$\therefore \quad x = 5$

$\therefore$ 5 students like both plain and chocolate milk.

A1 **C1**

b



A1 A1 A1ft **C3**

c P(likes only one type of milk) $= \dfrac{8+13}{30}$

$= \frac{21}{30}$

$= \frac{7}{10}$ or 0.7 A1 A1ft **C2**

[6 marks]

5 **a** Total number of events $= 12 + 15 + 11 + 7 + 5$

$= 50$

P(ticket costs more than \$60) $= \dfrac{11+7+5}{50}$

$= \frac{23}{50}$ or 0.46 A1 **C1**

b

Cost (\$)	*Number of events*	*Midpoint* (x)
20 - 39	12	29.5
40 - 59	15	49.5
60 - 79	11	69.5
80 - 99	7	89.5
100 - 119	5	109.5
Total	50	

Using technology, estimates are: mean $\approx$ \$60.70 A1

standard deviation $\approx$ \$25.35 A1 **C2**

c 0.722 standard deviations above the mean is $\$60.70 + 0.722 \times \$25.35 = \$79.00$ A1ft

The percentage of events less than \$79 $= \dfrac{12+15+11}{50}$

$= \frac{38}{50} \times 100\%$ M1

$= 76\%$ A1ft **C3**

[6 marks]

6 **a** 2000 USD $= 2000 \times 0.75$ GBP M1

$= 1500$ GBP A1 **C2**

b Amount remaining $= 1500 - 1200$ M1

$= 300$ GBP A1ft **C2**

300 GBP $= 300 \times \dfrac{1}{0.80}$ USD M1

$= 375$ USD A1ft **C2**

[6 marks]

7 **a** $u_1 r^6 = 320$ and $u_1 r^9 = 2560$ M1

$\therefore \quad \dfrac{u_1 r^9}{u_1 r^6} = \dfrac{2560}{320}$

$\therefore \quad r^3 = 8$

$\therefore \quad r = 2$ A1 **C2**

b $u_1 2^6 = 320$ M1

$\therefore \quad 64 u_1 = 320$

$\therefore \quad u_1 = 5$ A1ft **C2**

c $u_{20} = u_1 r^{19}$ M1

$= 5 \times 2^{19}$

$= 2\,621\,440$ A1ft **C2**

[6 marks]

8 **a** $F_v = C\left(1 + \dfrac{r}{100k}\right)^{kn}$

$= 20\,000 \times \left(1 + \frac{6.8}{1200}\right)^{48}$ M1 A1

$= \$26\,231.59$

The value after 48 months is \$26 231.59 A1 **C3**

b Increasing original investment by 3.2% p.a. gives

$20\,000(1.032)^4 = \$22\,685.52$ M1 A1 **C2**

c Real increase = future value − original value indexed

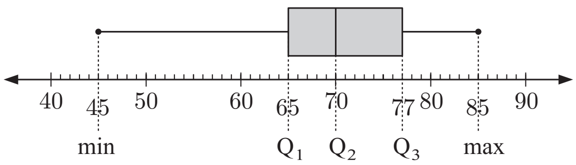
$= 26\,231.59 - 22\,685.52$

$= \$3546.07$ A1ft **C1**

[6 marks]

9 **a** $Q_3 = 77, \quad Q_1 = 65$ A1 A1 **C2**

b



$\text{IQR} = 77 - 65 = 12$ A1ft **C1**

c Minimum = 45 A1

Testing for outliers, $Q_1 - 1.5 \times \text{IQR}$

$= 65 - 1.5(12)$

$= 47$ M1

The minimum value is less than 47, so the minimum value is an outlier. A1ft **C3**

[6 marks]

10 **a** Profit = total sales − total cost

$\therefore$ for 100 boxes, profit $= 12.50(100) - (9.5(100) + 45)$ M1

$= \$255$ A1 **C2**

b The firm breaks even when $12.5x = 9.5x + 45$ M1

$\therefore \quad 3x = 45$

$\therefore \quad x = 15$

So, 15 boxes must be produced and sold to break even. A1 **C2**

c For the profit to be greater than \$1000, $\therefore \quad 12.50x - (9.5x + 45) > 1000$ M1

$\therefore \quad 3x - 45 > 1000$

$\therefore \quad x > 348.3$

So, 349 boxes must be produced and sold. A1 **C2**

[6 marks]

11 **a**

$p \wedge q$	$\neg(p \wedge q)$	$p \veebar q$	$\neg(p \wedge q) \vee q$	$(p \veebar q) \Rightarrow \neg(p \wedge q) \vee q$	
T	F	F	T	T	A1
F	T	T	T	T	A1
F	T	T	T	T	A1
F	T	F	T	T	A1 **C4**

b If Bozo does not have a red nose then Bozo is not a clown. A1 A1 **C2**

[6 marks]

12 A is $(-2, -3)$, B is $(1, 3)$

a The gradient of AB $= \dfrac{3-(-3)}{1-(-2)} = 2$ A1

$\therefore$ the equation of AB is $y = 2x + c$

Substituting $(1, 3)$ gives $3 = 2(1) + c$

$\therefore\ c = 1$ M1

The equation of AB is $y = 2x + 1$

or $2x - y + 1 = 0$ A1ft **C3**

b Midpoint of AB is $\left(\dfrac{-2+1}{2}, \dfrac{-3+3}{2}\right)$, or $\left(-\frac{1}{2}, 0\right)$ A1

The gradient of the perpendicular bisector is $-\frac{1}{2}$ $\{$as $2 \times -\frac{1}{2} = -1\}$ A1ft

$\therefore$ its equation is $y = -\frac{1}{2}x + c$

Substituting $\left(-\frac{1}{2}, 0\right)$ gives $0 = -\frac{1}{2}\left(-\frac{1}{2}\right) + c$

$\therefore\ c = -\frac{1}{4}$

$\therefore$ the equation of perpendicular bisector is $y = -\frac{1}{2}x - \frac{1}{4}$

or $4y = -2x - 1$

or $2x + 4y + 1 = 0$ A1ft **C3**

[6 marks]

13 **a** $f(x) = ax^2 + bx + d \quad \therefore\ f'(x) = 2ax + b$ A1 **C1**

b $f'(x) = 5x - 10$

Equating coefficients gives $2a = 5$ and $b = -10$

$\therefore\ a = 2.5$ and $b = -10$ A1ft A1ft **C2**

c $f'(x) = 0$ when $x = 2$ A1ft

Now $f(2) = 2.5 \times 2^2 - 10 \times 2 + d$

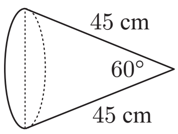
$= d - 10$

$\therefore\ d - 10 = -4$ M1

$\therefore\ d = 6$ A1ft **C3**

[6 marks]

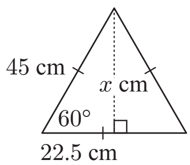
14 **a**



The triangle which bisects the cone is equilateral with sides 45 cm.

$\therefore$ the diameter of the megaphone is 45 cm. A1 **C1**

b



Let the height be x cm.

Now $\tan 60^o = \dfrac{x}{22.5}$ M1

$\therefore\ x \approx 39.0$ Alft

$\therefore$ height of cone is 39 cm.

Volume of cone $= \frac{1}{3}\pi r^2 h$

$\approx \frac{1}{3}\pi \times 22.5^2 \times 39$ M1 A1ft

$\approx 20\,700\ \text{cm}^3$ A1ft **C5**

[6 marks]

15 **a** **i** Amplitude $= \frac{1}{2}(\text{maximum} - \text{minimum})$

$= \frac{1}{2}(5 - 1)$

$= 2$ A1

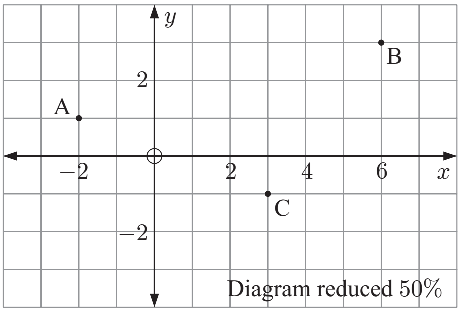
ii Period = length of 1 cycle $= 120^o$ A1 **C2**

b $y = a\cos(bx) + c$ where a = amplitude, $b = \dfrac{360^o}{\text{period}}$, c = vertical translation

So, $a = 2$, $b = \dfrac{360^o}{120^o} = 3$, and $c = 3$. A1 A2 A1 **C4**

[6 marks]

1 **a**



A1 A1 **2**

b **i** Gradient of BC $= \dfrac{-1-3}{3-6} = \dfrac{-4}{-3} = \frac{4}{3}$ M1 A1 (G2)

ii The opposite sides of a parallelogram are parallel, and parallel lines have the same gradient.
$\therefore$ gradient of AD = gradient of BC A1

iii Gradient of AD $= \dfrac{d-1}{-5--2} = \dfrac{4}{3}$ M1

$\therefore \quad \dfrac{d-1}{-3} = \dfrac{4}{3}$

$\therefore \quad d-1 = \frac{4}{3} \times -3$

$\therefore \quad d-1 = -4$

$\therefore \quad d = -3$ A1ft (G2) **5**

c **i** Length AB $= \sqrt{(6--2)^2 + (3-1)^2}$ M1 A1

$= \sqrt{68}$ units A1 (G2)

ii $\cos \widehat{ABC} = \dfrac{\sqrt{68}^2 + 5^2 - \sqrt{29}^2}{2 \times \sqrt{68} \times 5}$ M1 A1ft

$\therefore \quad \widehat{ABC} \approx 39.1^o$ A1ft (G2) **6**

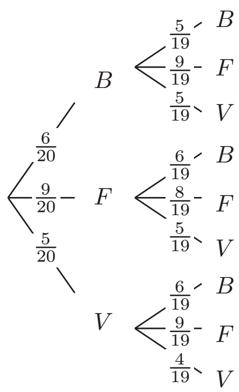
d Area ABCD = 2 × area triangle ABC

$= 2 \times \frac{1}{2} \times 5 \times \sqrt{68} \sin \widehat{ABC}$ M1 A1ft

$= 26$ units^2 A1ft (G2) **3**

[16 marks]

2 **a** Let B represent a basketball being chosen, F represent a football being chosen, and V represent a volleyball being chosen.



A4 **4**

b **i** P(two basketballs) = P(B then B)

$= \frac{6}{20} \times \frac{5}{19}$ M1

$= \frac{30}{380} \quad (\approx 0.0789)$ A1ft (G2)

ii P(a basketball and a football) = P(B then F) + P(F then B)

$= \frac{6}{20} \times \frac{9}{19} + \frac{9}{20} \times \frac{6}{19}$ M1 A1ft

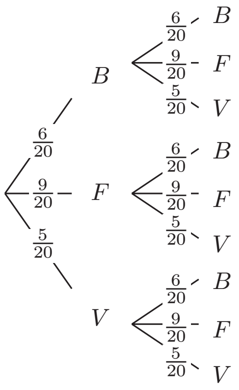
$= \frac{108}{380} \quad (\approx 0.284)$ A1ft (G2)

iii P(both balls are the same) = P(B then B) + P(F then F) + P(V then V)

$= \frac{6}{20} \times \frac{5}{19} + \frac{9}{20} \times \frac{8}{19} + \frac{5}{20} \times \frac{4}{19}$ M1 A1ft

$= \frac{122}{380} \quad (\approx 0.321)$ A1ft (G2) **8**

c With replacement:



i P(two volleyballs) $= \text{P}(V \text{ and } V)$

$= \frac{5}{20} \times \frac{5}{20}$ M1 A1ft

$= \frac{1}{4} \times \frac{1}{4}$

$= \frac{1}{16}$ A1ft (G2) **3**

ii $\text{P}(\text{both } B \,|\, \text{the two balls are the same})$

$$= \frac{\text{P}(B \text{ then } B)}{\text{P}(B \text{ then } B) + \text{P}(F \text{ then } F) + \text{P}(V \text{ then } V)}$$

$$= \frac{\frac{6}{20} \times \frac{6}{20}}{\frac{6}{20} \times \frac{6}{20} + \frac{9}{20} \times \frac{9}{20} + \frac{5}{20} \times \frac{5}{20}}$$ M1 A1ft

$= \frac{18}{71}$

≈ 0.254 A1ft (G2) **3**

[18 marks]

3 a i $T_P(0) = 61 \times (0.95)^0 + 18$

$= 61 + 18$

$= 79^o\text{C} \quad \therefore \ a = 79$ A1

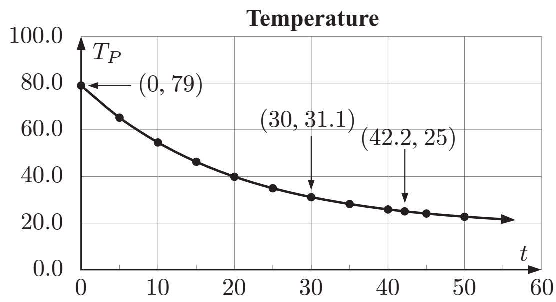
$T_P(30) = 61 \times (0.95)^{30} + 18$

$\approx 31.1^o\text{C} \quad \therefore \ b \approx 31.1$ A1 **2**

ii We need to solve $61 \times (0.95)^t + 18 = 25$ M1

So, $t \approx 42.2$ min {using technology} A1 (G2) **2**

b



(A1 for scale and labels,
A2 for points,
A1 for curve)

A1 A2ft A1 **4**

c i $T_F(0) = 53 \times (0.98)^0 + 18$ M1

$= 53 + 18$

$= 71^o\text{C}$ A1

ii As $0.95 < 0.98$, the T_P function decreases at a faster rate than the T_F function.

$\therefore$ heat is lost faster in the plastic cup. A1ft

iii We need to solve $53 \times (0.98)^t + 18 = 61 \times (0.95)^t + 18$ M1

$\therefore \quad 53 \times (0.98)^t = 61 \times (0.95)^t$

Using technology, $t \approx 4.52$ A1

So it takes about 4.5 minutes for the temperatures in each cup to be equal. A1 (G3) **6**

d In the long term, $(0.95)^t$ and $(0.98)^t$ both decrease to almost zero.

So, $T_P(t)$ and $T_F(t)$ both approach 18^oC. A2 **2**

[16 marks]

4 a i (1) $s_{xy} = 93.7$ so

$$r = \frac{s_{xy}}{s_x s_y} = \frac{93.7}{4.78 \times 21.4}$$ M1 A1

≈ 0.916 A1 (G2)

(2) A strong positive relationship exists between the maximum temperature and the number of people attending the swimming pool. A1ft A1ft **5**

ii The regression line is $y - \overline{y} = \dfrac{s_{xy}}{{s_x}^2}(x - \overline{x})$

$\therefore \quad y - 87.3 = \dfrac{93.7}{4.78^2}(x - 29.9)$ M1 A1

$\therefore \quad y - 87.3 \approx 4.101(x - 29.9)$

$\therefore \quad y \approx 4.101x - 122.62 + 87.3$

$\therefore \quad y \approx 4.10x - 35.32$

So, attendance $\approx 4.10 \times$ maximum temperature -35.32 M1 A1 (G3) **4**

iii **(1)** **i** When $x = 20$,

$y \approx 4.10 \times 20 - 35.32$ M1

$\therefore \quad y \approx 47$ people A1ft (G2)

ii When $x = 40$,

$y \approx 4.10 \times 40 - 35.32$

$\therefore \quad y \approx 129$ people A1ft

(2) The estimate at 20°C is more reliable as it is an interpolated value. A1ft R1ft **5**

iv Since the value of r is high, the manager's plan seems sensible. A1ft R1ft **2**

b **i** H_0: Attendance by gender is independent of the maximum temperature. A1 **1**

ii $\chi^2_{calc} \approx 2.79$ and p-value ≈ 0.0950 G2 **2**

iii As p-value > 0.05 we *do not* reject H_0. R1ft A1ft **2**

[21 marks]

5 **a** $f(x) = 3x^3 - 4x + 5$

i $f(1) = 3(1)^3 - 4(1) + 5$ M1

$= 4$ A1 (G2) **2**

ii $f'(x) = 9x^2 - 4$ A1 A1 **2**

iii $f'(1) = 9 - 4 = 5 \quad \therefore$ the gradient at $x = 1$ is 5 M1 A1ft (G2) **2**

iv At (1, 4) the tangent has gradient 5.

$\therefore$ its equation is $y - 4 = 5(x - 1)$ M1

or $y - 4 = 5x - 5$

or $y = 5x - 1$ A1ft (G1) **2**

v The tangent $y = 5x - 1$ meets $y = f(x)$ where $3x^3 - 4x + 5 = 5x - 1$

Using technology, $x = 1$ or -2

$\therefore$ they meet again at $(-2, -11)$. A1 A1ft **2**

b **i** Volume = length × width × height

$\therefore \quad V = x \times x \times y$

$= x^2y$ A2 **2**

ii $V = x^2\left(\dfrac{30\,000 - x^2}{2x}\right)$ M1

$= \frac{1}{2}x(30\,000 - x^2)$

$= 15\,000x - \frac{1}{2}x^3$ A1ft (G2) **2**

iii $\dfrac{dV}{dx} = 15\,000 - \frac{3}{2}x^2$ A1 A1ft **2**

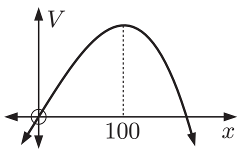
iv The maximum value occurs when $\dfrac{dV}{dx} = 0$ M1

$\therefore \quad \frac{3}{2}x^2 = 15\,000$

$\therefore \quad 3x^2 = 30\,000$

$\therefore \quad x^2 = 10\,000$

$\therefore \quad x = 100 \quad \{x > 0\}$ A1



$\therefore$ the maximum value of V is when $x = 100$. A1ft (G2) **3**

[19 marks]

SOLUTIONS TO SPECIMEN EXAMINATION B - PAPER 1

1 **a** **C** and **D** A1 A1 **C2**

b $0.0518 = 5.18 \times 10^{-2}$ A1 A1 **C2**