

- 4 30 students were asked the type of milk drinks they liked. 8 students said they liked plain milk only, 13 liked chocolate milk only, and 4 students did not like either.
- Find the number of students who liked both plain and chocolate milk.
 - Represent the information above on a Venn diagram.
 - Determine the probability of a student, chosen at random from this group, liking only one type of milk.

- 5 The frequency table below shows the price of tickets for various events at the Festival of Arts.

Cost (\$)	Number of events
20 - 39	12
40 - 59	15
60 - 79	11
80 - 99	7
100 - 119	5

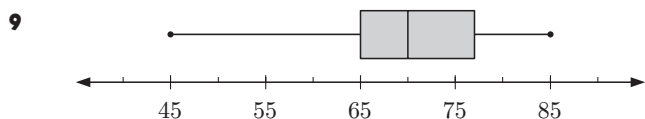
- Find the probability that a ticket for a randomly chosen event at the festival costs \$60 or more.
 - Find the mean and standard deviation for the price of tickets.
 - Find the percentage of events whose ticket price is less than 0.722 standard deviations above the mean.
- 6 Andy travels from New York to London. He can buy 1.00 USD for 0.80 GBP, and sell 1.00 USD for 0.75 GBP.
- If Andy converts 2000 USD into GBP, how many GBP does he receive?
 - If Andy spends 1200 GBP while in London, and converts the remainder back to USD, how many USD will he receive?

- 7 The seventh term of a geometric sequence is 320 and the tenth term is 2560. Find:

- the common ratio
- the first term
- the twentieth term.

- 8 James and Lesley invested \$20 000 in a fund which paid 6.8% per annum nominal interest, compounded monthly.

- Find the value of their investment after 48 months.
- The average rate of inflation during these 4 years was 3.2% per annum.
Increase the value of the original investment to account for inflation over the four years.
- The *real increase* for an investment is defined as the difference between the future value and the original value, indexed for inflation.
Calculate the real increase in the value of James and Lesley's investment.



For the box plot shown above:

- find the upper and lower quartiles
- calculate the interquartile range
- determine whether the minimum value is an outlier.

- 10 A company sells plastic boxes for \$12.50 each. It estimates the total cost of producing x boxes as $\$(9.5x + 45)$. Calculate:

- the profit made when 100 boxes are produced and sold

- the number of boxes which must be produced and sold for the firm to 'break even'
- the number of boxes which must be produced and sold to make a profit of \$1000.

- 11 The truth table below shows some truth values for the statement $(p \vee q) \Rightarrow \neg(p \wedge q) \vee q$:

$p \wedge q$	$\neg(p \wedge q)$	$p \vee q$	$\neg(p \wedge q) \vee q$	$(p \vee q) \Rightarrow \neg(p \wedge q) \vee q$
T	F	F		
F	T	T	T	T
F	T	T	T	T
F	T			T

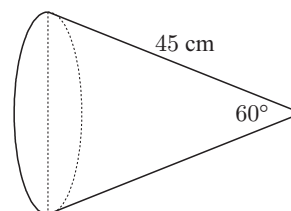
- Fill in the missing truth values on the table.
- Write down the contrapositive of the proposition:
If Bozo is a clown then Bozo has a red nose.

- 12
- Find the equation of the line joining the points $A(-2, -3)$ and $B(1, 3)$. Give your answer in the form $ax + by + d = 0$ where $a, b, d \in \mathbb{Z}$.
 - Find the equation of the perpendicular bisector of AB.

- 13 A function is defined as $f(x) = ax^2 + bx + d$ where a, b and d are integers.

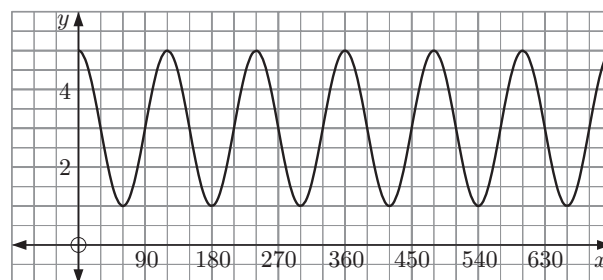
- Find an expression for $f'(x)$.
- If $f'(x) = 5x - 10$, find the values of a and b .
- The minimum value of $f(x)$ is -4 .
Determine the x -coordinate of the minimum value of $f(x)$, and hence find the value of d .

- 14 A megaphone in the shape of a cone has vertical angle 60° and a slant height of 45 cm as shown in the diagram below.



- Determine the diameter of the megaphone.
- Determine the volume of the megaphone.

- 15 The diagram shows part of the graph of a cosine function.



- Using the graph, find:
 - the amplitude
 - the period of the function.
- If the graph has equation $y = a \cos(bx) + c$ state the values of a, b and c .

1 The coordinates of the vertices of a parallelogram are $A(-2, 1)$, $B(6, 3)$, $C(3, -1)$, and $D(-5, d)$. AC is a diagonal of the parallelogram.

- a Using a scale of 1 cm to represent 1 unit on both axes, plot the points A , B and C . (2 marks)
- b
- Find the gradient of the line through B and C .
 - Explain why the gradient of AD is the same as the gradient of BC .
 - Find the value of d . (5 marks)
- c The length of AC is $\sqrt{29}$ units, and the length of BC is 5 units.
- Find the length of AB .
 - Find the measure of angle ABC . (6 marks)
- d Find the area of parallelogram $ABCD$. (3 marks)

2 A sports locker at school contains 6 basketballs, 9 footballs, and 5 volleyballs. During a sports lesson, a teacher chooses two balls from the locker at random, without replacement.

- a Draw a tree diagram showing all the possible outcomes. Write the probabilities for each branch on the diagram. (4 marks)
- b Find the probability that the teacher chooses:
- two basketballs
 - one basketball and one football
 - both balls the same. (8 marks)
- c The two balls are replaced at the end of the lesson. During the next lesson, another teacher chooses a ball at random from the locker, replaces it, and then chooses a second ball. Find the probability that the balls chosen are:
- two volleyballs
 - both basketballs, given that the two balls are the same. (6 marks)

3 The temperature of a cup of coffee in a plastic cup is modelled by $T_P(t) = 61 \times (0.95)^t + 18$ °C, where t is the time in minutes after it is poured.

- a
- Calculate the values of a and b in the table below.

Time t (min)	0	5	10	15	20	25	30
Temp. (°C)	a	65.2	54.5	46.3	39.9	34.9	b

- Find the time it will take for the temperature to reach 25 °C. (4 marks)
- b On graph paper draw and label a graph representing this information. Use 1 cm for every two minutes on the horizontal axis and 1 cm for every 10 °C on the vertical axis. (4 marks)
- c A china cup is used for a new cup of coffee. The temperature of coffee in this cup is given by $T_F(t) = 53 \times (0.98)^t + 18$ °C.
- Determine the initial temperature of the new cup of coffee.
 - Comment on the rate of heat loss of the china cup compared to the original plastic cup.
 - Using your graphics calculator, find the time it takes before the temperatures in the cups are the same. Give your answer correct to the nearest tenth of a minute. (6 marks)

- d In the longer term, what temperature will each cup of coffee approach? (2 marks)

4 a The following table shows the number of people using a public swimming pool in a particular month. The maximum daily temperatures vary from 18 °C to 35 °C.

Max. temperature (°C)		Daily attendance	
Mean	29.9	Mean	87.3
Standard deviation	4.78	Standard deviation	21.4

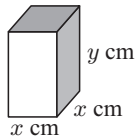
The covariance for temperature and attendance is 93.7.

- (1) Determine the coefficient of correlation r for this data.
 - (2) Describe the relationship between the maximum temperature and attendance at the swimming pool. (5 marks)
- ii Find the equation of the linear regression line for attendance as a function of temperature. (4 marks)
- iii Use the equation of linear regression to estimate the number of people attending the pool on a day when the maximum temperature is:
- 20 °C
 - 40 °C.
- (2) Which of the estimates in **iii (1)** is the more reliable? Give a reason for your answer. (5 marks)
- iv The manager of the pool plans to use the forecast temperature to determine the number of staff to be employed each day. Does the manager's plan seem sensible? Justify your answer. (2 marks)
- b The manager of the pool records the gender of the swimmers attending the pool each day. She believes that the maximum temperature on any day causes a different attendance pattern for males and females. Given the following information about average daily attendance and temperature, conduct a chi-squared test at the 5% significance level to determine if the manager is right.

	Temp < 30 °C	Temp ≥ 30 °C
Male	36	47
Female	51	40

- Write down a suitable hypothesis for the chi-squared test. (1 mark)
 - Find the p -value of the chi-squared statistic for this data. (2 marks)
 - What conclusion can be drawn from the test? Justify your answer. (2 marks)
- 5 a Consider the function $f(x) = 3x^3 - 4x + 5$.
- Find $f(1)$. (2 marks)
 - Calculate $f'(x)$. (2 marks)
 - Find the gradient of the tangent at $x = 1$. (2 marks)
 - Determine algebraically the equation of the tangent to the curve given by $f(x) = 3x^3 - 4x + 5$ at the point where $x = 1$. (2 marks)
 - The tangent to the curve at $x = 1$ intersects $f(x)$ at one other point. Using your graphic display calculator, find the coordinates of this point of intersection. (2 marks)

- b** A rectangular box has a square base as shown.



- i** Write down an expression for the volume V of the box.

(2 marks)

- ii** Given $y = \frac{30\,000 - x^2}{2x}$, write the volume in terms of x only.

(2 marks)

- iii** Find $\frac{dV}{dx}$.

(2 marks)

- iv** Hence, find the value of x which maximises the volume of the box.

(3 marks)

SPECIMEN EXAMINATION B PAPER 1

- 1** The number 0.051 762 is rounded to 0.0518.

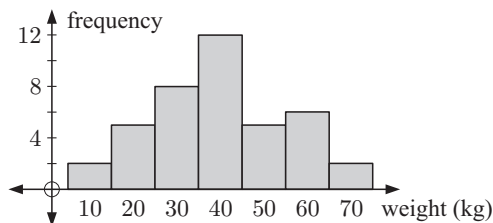
- a** State which *two* of the following are accurate descriptions of the rounding:

- A** correct to 3 decimal places
B correct to 4 significant figures
C correct to 3 significant figures
D correct to the nearest ten-thousandth.

- b** Write 0.0518 in the form $a \times 10^k$ where $0 \leq a < 10$, $k \in \mathbb{Z}$.

- c** Calculate the percentage error in the rounding.

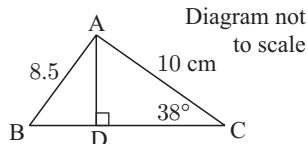
- 2** The histogram shows the weight of sheep (to the nearest 10 kg).



- a** Find the number of sheep that were weighed.
b Estimate the mean weight for these sheep.
c The farmer sends to the market all sheep whose weights are more than 50% above the mean.

Determine the percentage of these sheep that will be delivered.

- 3** Triangle ABC is shown in the figure with $AC = 10$ cm and $AB = 8.5$ cm. Angle $CDA = 90^\circ$ and angle $ACD = 38^\circ$.



Calculate the length of: **a** DC **b** BD.

- 4** The cost of producing x pairs of jeans is $C(x) = 15.6x + 245$ euros. Each pair of jeans can be sold for €42.50. Find:

- a** the total revenue obtained by selling 22 pairs of jeans
b the cost of producing 22 pairs of jeans
c the profit made by producing and selling 22 pairs of jeans
d the profit per pair of jeans in this case.

- 5** P and Q are subsets of a universal set U such that:

$$U = \{1 \leq x \leq 13, x \in \mathbb{Z}\},$$

$$P = \{\text{prime numbers between 1 and 13 inclusive}\},$$

$$Q = \{\text{factors of 24}\}.$$

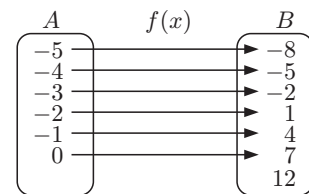
List the members of sets: **a** P **b** Q **c** $(P \cup Q)'$

- 6** A box contains 10 wooden shapes. There are 5 triangles, 4 rectangles, and 1 rhombus. 2 shapes are chosen at random from the box without replacement.

Calculate the probability that:

- a** both are triangles
b one of the chosen shapes is the rhombus
c the first chosen is a rectangle, given that the second is also a rectangle.

- 7** The diagram shows a function $f(x)$, mapping members of the set A to members of set B .



- a** Write down an inequality for x describing the domain.
b Using set notation, write down all members of the range of $f(x)$.
c Write down the equation of the function $f(x)$ in the form $y = mx + c$.

- 8** Jacinta won a prize of \$5000. She spent half the money and placed the remainder in an investment account which paid simple interest of 7.5% per annum.

- a** Calculate the amount in the account after $3\frac{1}{2}$ years.
b Find the rate of simple interest that would have returned an amount of \$3400 in the account after $3\frac{1}{2}$ years.

- 9** **a** Differentiate the function $y = 2x^3 - 3x^2 - 264x + 13$.
b Hence, find the x -coordinates of the points where the gradient to the function is equal to -12 .

- 10** Eddie purchases 4 reams of paper and 3 pens for a total cost of £19.

Let the price of a ream of paper be $\mathcal{L}r$ and the price of a pen be $\mathcal{L}p$.

- a** Write an equation in r and p that represents Eddie's purchase.
b If the cost of 2 pens was deducted from the cost of 3 reams of paper, the total amount would be £10. Write a second equation in r and p that represents this information.
c Solve the two equations you have written simultaneously.
d Hence determine the cost of purchasing 5 reams of paper and 5 pens.

- 11** The table below shows the different activities chosen by a class of final year students on their end of year expedition.

	Climbing	Swimming	Mountain Biking
Female	9	18	8
Male	15	16	24