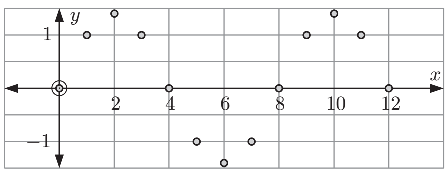
Chapter 10

ADVANCED TRIGONOMETRY

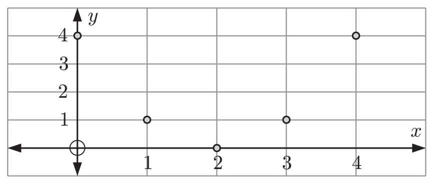
EXERCISE 10A

1 **a**



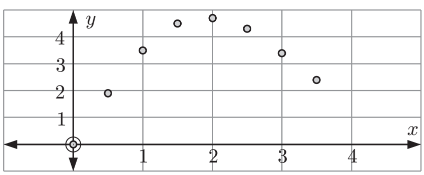
Data exhibits periodic behaviour.

b



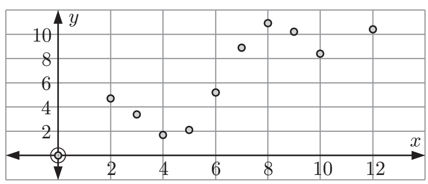
Not enough information to say data is periodic. It may in fact be quadratic.

c



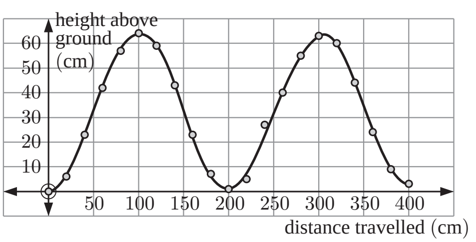
Not enough information to say data is periodic. It may in fact be quadratic.

d



Not enough information to say data is periodic.

2 **a**



b The data is periodic.

i The minimum value from the table is 0 and the maximum value is 64.
So, the principal axis is $y \approx \frac{0+64}{2}$,
$\therefore\ y \approx 32$.

ii The maximum value is ≈ 64 cm.

iii The period is ≈ 200 cm.

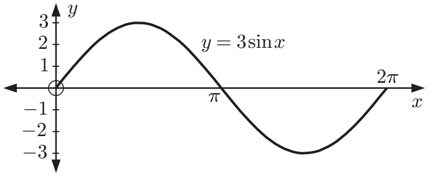
iv The amplitude is ≈ 32 cm.

c A curve can be fitted to the data as the distance travelled is continuous.

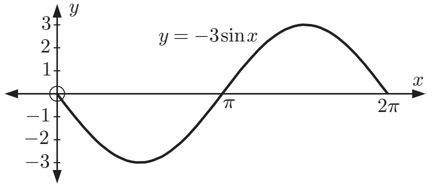
3 **a** periodic **b** periodic **c** periodic **d** not periodic **e** periodic **f** periodic

EXERCISE 10B.1

1 **a** $y = 3\sin x$
has amplitude 3 and period $\frac{2\pi}{1} = 2\pi$
When $x = 0$, $y = 0$.

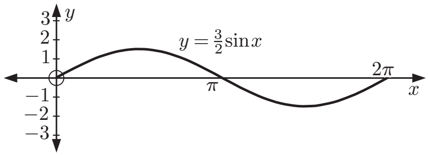


b $y = -3\sin x$
has amplitude $|-3| = 3$
and period $\frac{2\pi}{1} = 2\pi$.
When $x = 0$, $y = 0$.

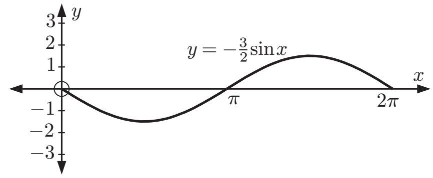


It is the reflection of $y = 3\sin x$ in the x-axis.

c $y = \frac{3}{2}\sin x$
has amplitude $\frac{3}{2}$ and period $\frac{2\pi}{1} = 2\pi$.
When $x = 0$, $y = 0$.

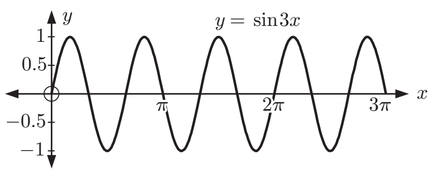


d $y = -\frac{3}{2}\sin x$
has amplitude $\left|-\frac{3}{2}\right| = \frac{3}{2}$
and period $\frac{2\pi}{1} = 2\pi$.

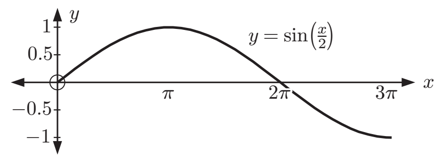


It is the reflection of $y = \frac{3}{2}\sin x$ in the x-axis.

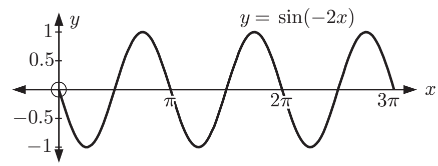
2 **a** $y = \sin 3x$
has amplitude 1 and period $\frac{2\pi}{3}$.
When $x = 0$, $y = 0$.



b $y = \sin\left(\frac{x}{2}\right)$
has amplitude 1 and period $\dfrac{2\pi}{\frac{1}{2}} = 4\pi$.
When $x = 0$, $y = 0$.



c $y = \sin(-2x)$
has amplitude 1 and period $\dfrac{2\pi}{|-2|} = \pi$.
When $x = 0$, $y = 0$.



It is the reflection of $y = \sin 2x$ in the y-axis.

3 **a** period $= \dfrac{2\pi}{4} = \frac{\pi}{2}$

b period $= \dfrac{2\pi}{|-4|} = \frac{\pi}{2}$

c period $= \dfrac{2\pi}{\left(\frac{1}{3}\right)} = 6\pi$

d period $= \dfrac{2\pi}{0.6} = \frac{20\pi}{6} = \frac{10\pi}{3}$

4 **a** $\dfrac{2\pi}{b} = 5\pi$ $\therefore\ b = \frac{2}{5}$

b $\dfrac{2\pi}{b} = \dfrac{2\pi}{3}$ $\therefore\ b = 3$

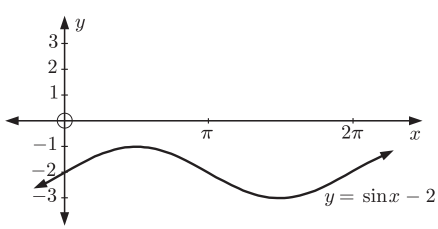
c $\dfrac{2\pi}{b} = 12\pi$ $\therefore\ b = \frac{1}{6}$

d $\dfrac{2\pi}{b} = 4$ $\therefore\ b = \frac{\pi}{2}$

e $\dfrac{2\pi}{b} = 100$ $\therefore\ b = \frac{2\pi}{100} = \frac{\pi}{50}$

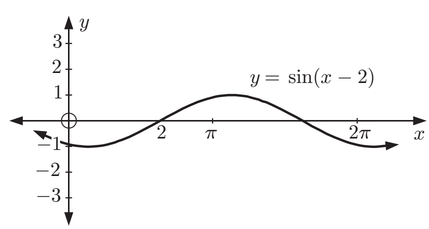
EXERCISE 10B.2

1 **a**



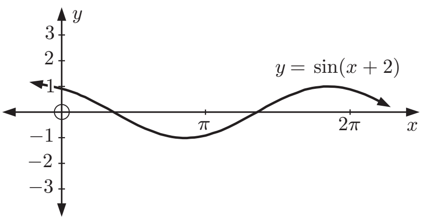
This is the graph of $y = \sin x$ translated by $\binom{0}{-2}$.

b



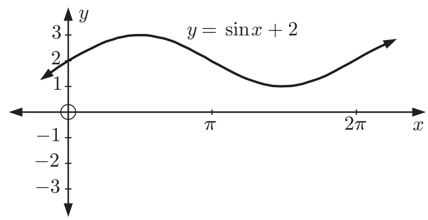
This is the graph of $y = \sin x$ translated by $\binom{2}{0}$.

c



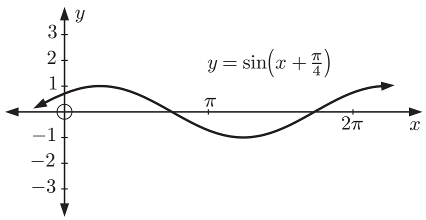
This is the graph of $y = \sin x$ translated by $\begin{pmatrix} -2 \\ 0 \end{pmatrix}$.

d



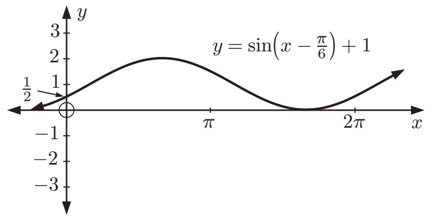
This is the graph of $y = \sin x$ translated by $\begin{pmatrix} 0 \\ 2 \end{pmatrix}$.

e



This is the graph of $y = \sin x$ translated by $\begin{pmatrix} -\frac{\pi}{4} \\ 0 \end{pmatrix}$.

f



This is the graph of $y = \sin x$ translated by $\begin{pmatrix} \frac{\pi}{6} \\ 1 \end{pmatrix}$.

2 **a** period $= \dfrac{2\pi}{5} = \frac{2\pi}{5}$ **b** period $= \dfrac{2\pi}{\left(\frac{1}{4}\right)} = 8\pi$ **c** period $= \dfrac{2\pi}{|-2|} = \pi$

3 **a** $\dfrac{2\pi}{b} = 3\pi$ $\therefore\ b = \frac{2}{3}$ **b** $\dfrac{2\pi}{b} = \dfrac{\pi}{10}$ $\therefore\ b = 20$ **c** $\dfrac{2\pi}{b} = 100\pi$ $\therefore\ b = \frac{2}{100} = \frac{1}{50}$ **d** $\dfrac{2\pi}{b} = 50$ $\therefore\ b = \frac{2\pi}{50} = \frac{\pi}{25}$

4 **a** A translation of $\begin{pmatrix} 0 \\ -1 \end{pmatrix}$, or vertically down 1 unit.

b A translation of $\begin{pmatrix} \frac{\pi}{4} \\ 0 \end{pmatrix}$, or horizontally $\frac{\pi}{4}$ units right.

c A vertical stretch of factor 2.

d A horizontal stretch of factor $\frac{1}{4}$.

e A vertical stretch of factor $\frac{1}{2}$.

f A horizontal stretch of factor 4.

g A reflection in the x-axis.

h A translation of $\begin{pmatrix} -2 \\ -3 \end{pmatrix}$.

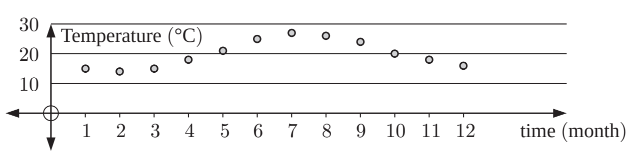
i A vertical stretch of factor 2 followed by a horizontal stretch of factor $\frac{1}{3}$.

j A translation of $\begin{pmatrix} \frac{\pi}{3} \\ 2 \end{pmatrix}$.

EXERCISE 10C

1 **a**

Month, t	1	2	3	4	5	6	7	8	9	10	11	12
Temp, T	15	14	15	18	21	25	27	26	24	20	18	16



The period is 12 months so $\dfrac{2\pi}{b} = 12$ $\therefore\ b = \frac{\pi}{6}$ {assuming $b > 0$}.

Amplitude, $a \approx \dfrac{\text{max.} - \text{min.}}{2} \approx \dfrac{27 - 14}{2} \approx 6.5$

As the principal axis is midway between min. and max., then $d \approx \dfrac{27 + 14}{2} \approx 20.5$

When T is 20.5 (midway between min. and max.)

$c \approx \dfrac{2+7}{2} \approx 4.5$ {average of t values}

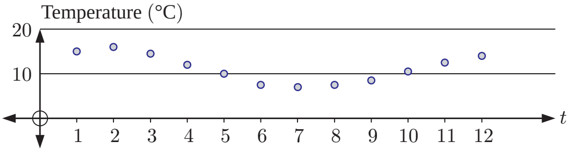
$\therefore \quad T \approx 6.5 \sin \frac{\pi}{6}(t - 4.5) + 20.5$ where $\frac{\pi}{6} \approx 0.524$.

b Using technology, $T \approx 6.14 \sin(0.575t - 2.70) + 20.4$

$\therefore \quad T \approx 6.14 \sin 0.575(t - 4.70) + 20.4$

2 **a**

Month, t	1	2	3	4	5	6	7	8	9	10	11	12
Temp, T	15	16	$14\frac{1}{2}$	12	10	$7\frac{1}{2}$	7	$7\frac{1}{2}$	$8\frac{1}{2}$	$10\frac{1}{2}$	$12\frac{1}{2}$	14



The period is $\dfrac{2\pi}{b} = 12 \quad \therefore \quad b = \frac{\pi}{6} \quad \{b > 0\}$

Amplitude, $a \approx \dfrac{\text{max.} - \text{min.}}{2} \approx \dfrac{16 - 7}{2} \approx 4.5$

As the principal axis is midway between min. and max. then $d \approx \dfrac{16 + 7}{2} \approx 11.5$

At min., $t = 7$ and at max., $t = 2 + 12 = 14 \quad \therefore \quad c \approx \dfrac{7 + 14}{2} \approx 10.5$

So, $T \approx 4.5 \sin \frac{\pi}{6}(t - 10.5) + 11.5$

b Using technology, $T \approx 4.29 \sin(0.533t + 0.769) + 11.2$

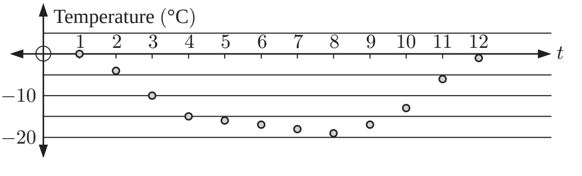
$\therefore \quad T \approx 4.29 \sin 0.533(t + 1.44) + 11.2$

Note: (1) $\frac{\pi}{6} \approx 0.524$ ✓

(2) $1.44 - (-10.5) = 11.94$
≈ 12

3

Month, t	1	2	3	4	5	6	7	8	9	10	11	12
Temp, T	0	-4	-10	-15	-16	-17	-18	-19	-17	-13	-6	-1



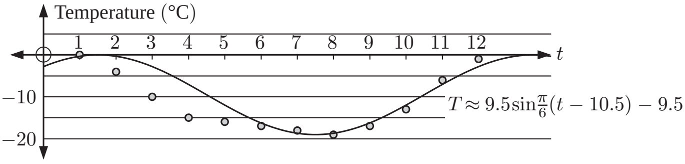
The period is $\dfrac{2\pi}{b} = 12 \quad \therefore \quad b = \frac{\pi}{6} \quad \{b > 0\}$

Amplitude, $a \approx \dfrac{\text{max.} - \text{min.}}{2} \approx \dfrac{0 - (-19)}{2} \approx 9.5$

$d \approx \dfrac{\text{max.} + \text{min.}}{2} \approx \dfrac{0 + (-19)}{2} \approx -9.5$

At min., $t = 8$ and at max., $t = 1 + 12 = 13 \quad \therefore \quad c \approx \dfrac{8 + 13}{2} \approx 10.5$

So, $T \approx 9.5 \sin \frac{\pi}{6}(t - 10.5) - 9.5$



The model is not very appropriate.

4 **a** For the model $H = a \sin b(t - c) + d$

$$\text{period} = \frac{2\pi}{b} = 12.4 \text{ hours} \quad \therefore \quad b = \frac{2\pi}{12.4} \approx 0.507$$

We let the principal axis be 0, so $d = 0$

$\therefore$ the amplitude $a = 7$, so the min. is -7, and the max. is $+7$

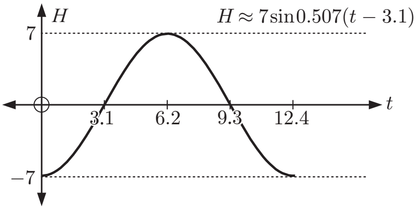
Let $t = 0$ correspond to 'low tide' $\therefore$ $t = 6.2$ corresponds to 'high tide'

$\therefore \quad c = \dfrac{0 + 6.2}{2} = 3.1$

So, $H \approx 7 \sin 0.507(t - 3.1) + 0$

$\therefore \quad H \approx 7 \sin 0.507(t - 3.1)$

b



5 Let the model be $H = a \sin b(t - c) + d$ metres

When $t = 0$, $H = 2$ (↑ min.) and when $t = 50$, $H = 22$ (↑ max.)

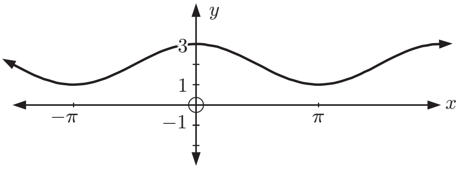
$$\text{period} = \frac{2\pi}{b} = 100 \quad \therefore \quad b = \frac{2\pi}{100} = \frac{\pi}{50}$$

$a = 10$ {from the diagram} $\quad d = \dfrac{\text{max.} + \text{min.}}{2} = \dfrac{22 + 2}{2} = 12$

$c = \dfrac{0 + 50}{2} = 25$ {values of t at min. and max.} $\quad \therefore \quad H = 10 \sin \dfrac{\pi}{50}(t - 25) + 12$

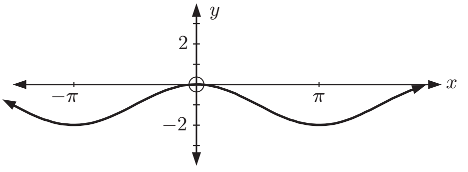
EXERCISE 10D

1 **a** $y = \cos x + 2$



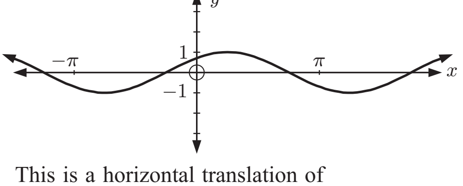
This is a vertical translation of $y = \cos x$ through $\binom{0}{2}$.

b $y = \cos x - 1$



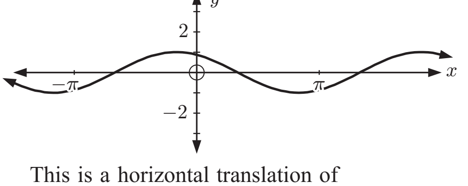
This is a vertical translation of $y = \cos x$ through $\binom{0}{-1}$.

c $y = \cos(x - \frac{\pi}{4})$



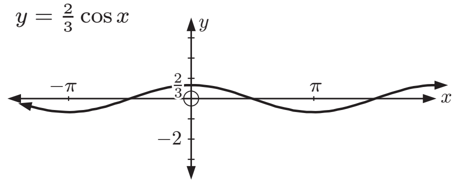
This is a horizontal translation of $y = \cos x$ through $\begin{pmatrix} \frac{\pi}{4} \\ 0 \end{pmatrix}$.

d $y = \cos(x + \frac{\pi}{6})$



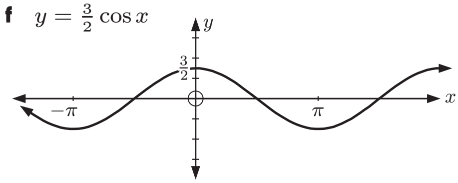
This is a horizontal translation of $y = \cos x$ through $\begin{pmatrix} -\frac{\pi}{6} \\ 0 \end{pmatrix}$.

e $y = \frac{2}{3}\cos x$



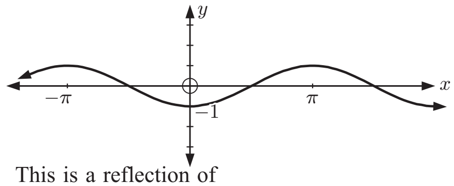
This is a vertical stretch of $y = \cos x$ with factor $\frac{2}{3}$.

f $y = \frac{3}{2}\cos x$



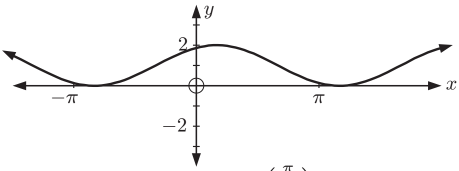
This is a vertical stretch of $y = \cos x$ with factor $\frac{3}{2}$.

g $y = -\cos x$



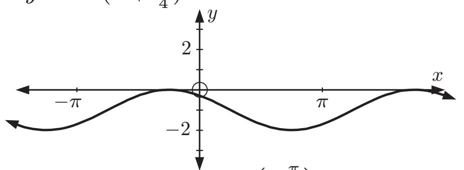
This is a reflection of $y = \cos x$ in the x-axis.

h $y = \cos(x - \frac{\pi}{6}) + 1$



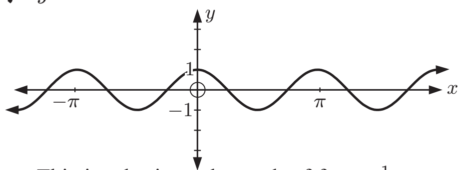
This is a translation of $\begin{pmatrix} \frac{\pi}{6} \\ 1 \end{pmatrix}$.

i $y = \cos(x + \frac{\pi}{4}) - 1$



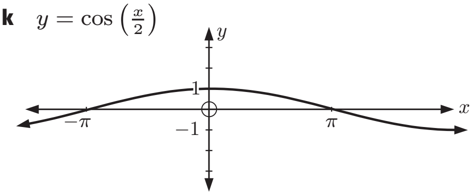
This is a translation of $\begin{pmatrix} -\frac{\pi}{4} \\ -1 \end{pmatrix}$.

j $y = \cos 2x$



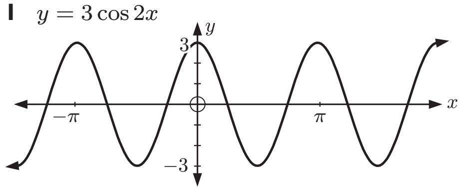
This is a horizontal stretch of factor $\frac{1}{2}$.

k $y = \cos\left(\frac{x}{2}\right)$



This is a horizontal stretch of factor 2.

l $y = 3\cos 2x$



This is a horizontal stretch of factor $\frac{1}{2}$ followed by a vertical stretch of factor 3.

2 **a** period $= \frac{2\pi}{3}$ **b** period $= \dfrac{2\pi}{\frac{1}{3}} = 6\pi$ **c** period $= \dfrac{2\pi}{\frac{\pi}{50}} = 100$

3 a controls the amplitude {amplitude $= |a|$}. b controls the period {period $= \dfrac{2\pi}{|b|}$}.
c controls the horizontal translation. d controls the vertical translation.

4 **a** If $y = a\cos b(x - c) + d$, then $a = 2$, $\pi = \dfrac{2\pi}{b}$ $\therefore$ $b = 2$
c and d are 0 as there is no horizontal or vertical shift. $\therefore$ $y = 2\cos(2x)$

b If $y = a\cos b(x - c) + d$, then $a = 1$, $4\pi = \dfrac{2\pi}{b}$ $\therefore$ $b = \frac{1}{2}$
A vertical shift of 2 units, no horizontal shift $\therefore$ $d = 2$, $c = 0$.
So, $y = \cos(\frac{1}{2}x) + 2$ or $y = \cos\left(\frac{x}{2}\right) + 2$.

c If $y = a\cos b(x - c) + d$, then $a = -5$, $6 = \dfrac{2\pi}{b}$ $\therefore$ $b = \frac{\pi}{3}$
$c = d = 0$ {as there is no translation} $\therefore$ $y = -5\cos\left(\frac{\pi}{3}x\right)$