Chapter 16

LINES AND PLANES IN SPACE

EXERCISE 16A.1

1 **a** **i** $\binom{x}{y} = \binom{3}{-4} + t\binom{1}{4}$ **ii** $x = 3 + t, \quad y = -4 + 4t, \quad t \in \mathbb{R}$

b **i** $\binom{x}{y} = \binom{5}{2} + t\binom{-8}{2}$ **ii** $x = 5 - 8t, \quad y = 2 + 2t, \quad t \in \mathbb{R}$

c **i** $\binom{x}{y} = \binom{-6}{0} + t\binom{3}{7}$ **ii** $x = -6 + 3t, \quad y = 7t, \quad t \in \mathbb{R}$

d **i** $\binom{x}{y} = \binom{-1}{11} + t\binom{-2}{1}$ **ii** $x = -1 - 2t, \quad y = 11 + t, \quad t \in \mathbb{R}$

2 $x = -1 + 2\lambda, \ y = 4 - \lambda, \quad \lambda \in \mathbb{R}$

When $\lambda = 0, \ x = -1 + 2(0) = -1$ and $y = 4 - 0 = 4$ $\quad \therefore$ the point is $(-1, 4)$.

When $\lambda = 1, \ x = -1 + 2(1) = 1$ and $y = 4 - 1 = 3$ $\quad \therefore$ the point is $(1, 3)$.

When $\lambda = 3, \ x = -1 + 2(3) = 5$ and $y = 4 - 3 = 1$ $\quad \therefore$ the point is $(5, 1)$.

When $\lambda = -1, \ x = -1 + 2(-1) = -3$ and $y = 4 - -1 = 5$ $\quad \therefore$ the point is $(-3, 5)$.

When $\lambda = -4, \ x = -1 + 2(-4) = -9$ and $y = 4 - -4 = 8$ $\quad \therefore$ the point is $(-9, 8)$.

3 **a** If $t + 2 = 3$ and $1 - 3t = -2$, we get $t = 1$ and $-3t = -3 \quad \therefore \ t = 1$

Since $t = 1$ in each case, $(3, -2)$ lies on the line.

If $t + 2 = 0$ and $1 - 3t = 6$, we get $t = -2$ and $-3t = 5 \quad \therefore \ t = -\frac{5}{3}$

$\therefore$ $(0, 6)$ does not lie on the line.

b If $(k, 4)$ lies on $x = 1 - 2t, \ \ y = 1 + t$, then

$k = 1 - 2t$ and $4 = 1 + t$

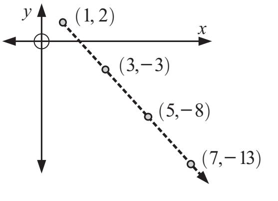
$\therefore \ t = 3$

and $k = 1 - 6 = -5$

4 **a** $x(0) = 1$ and $y(0) = 2$,

$\therefore$ the initial position is $(1, 2)$

b



c In 1 second, the x-step is 2 and y-step is -5, which is a distance of $\sqrt{2^2 + (-5)^2} = \sqrt{29}$ cm

$\therefore$ the speed is $\sqrt{29}$ cm s^{-1}.

EXERCISE 16A.2

1 **a** The vector equation is $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 3 \\ -7 \end{pmatrix} + t\begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix}, \quad t \in \mathbb{R}$

b The vector equation is $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix} + t\begin{pmatrix} 1 \\ 1 \\ -2 \end{pmatrix}, \quad t \in \mathbb{R}$

c Since the line is parallel to the X-axis, it has direction vector $\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$

$\therefore$ the vector equation is $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -2 \\ 2 \\ 1 \end{pmatrix} + t\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \quad t \in \mathbb{R}$

2 **a** The parametric equations are:
$x = 5 + (-1)t,\ \ y = 2 + 2t,\ \ z = -1 + 6t$
$\therefore\ \ x = 5 - t,\ \ y = 2 + 2t,\ \ z = -1 + 6t,\ \ t \in \mathbb{R}$

b The parametric equations are:
$x = 0 + 2t,\ \ y = 2 + (-1)t,\ \ z = -1 + 3t$
$\therefore\ \ x = 2t,\ \ y = 2 - t,\ \ z = -1 + 3t,\ \ t \in \mathbb{R}$

c Since the line is perpendicular to the XOY plane, it has direction vector $\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$.
$\therefore$ the parametric equations are: $x = 3 + 0t,\ \ y = 2 + 0t,\ \ z = -1 + 1t$
$\therefore\ \ x = 3,\ \ y = 2,\ \ z = -1 + t,\ \ t \in \mathbb{R}$

3 **a** $\overrightarrow{AB} = \begin{pmatrix} -1-1 \\ 3-2 \\ 2-1 \end{pmatrix} = \begin{pmatrix} -2 \\ 1 \\ 1 \end{pmatrix} \quad \therefore\ x = 1 - 2t,\quad y = 2 + t,\quad z = 1 + t,\quad t \in \mathbb{R}$

b $\overrightarrow{CD} = \begin{pmatrix} 3-0 \\ 1-1 \\ -1-3 \end{pmatrix} = \begin{pmatrix} 3 \\ 0 \\ -4 \end{pmatrix} \quad \therefore\ x = 3t,\quad y = 1,\quad z = 3 - 4t,\quad t \in \mathbb{R}$

c $\overrightarrow{EF} = \begin{pmatrix} 1-1 \\ -1-2 \\ 5-5 \end{pmatrix} = \begin{pmatrix} 0 \\ -3 \\ 0 \end{pmatrix} \quad \therefore\ x = 1,\quad y = 2 - 3t,\quad z = 5,\quad t \in \mathbb{R}$

d $\overrightarrow{GH} = \begin{pmatrix} 5-0 \\ -1-1 \\ 3--1 \end{pmatrix} = \begin{pmatrix} 5 \\ -2 \\ 4 \end{pmatrix} \quad \therefore\ x = 5t,\quad y = 1 - 2t,\quad z = -1 + 4t,\quad t \in \mathbb{R}$

4 Given $x = 1 - \lambda,\ \ y = 3 + \lambda,\ \ z = 3 - 2\lambda$:

a The line meets the XOY plane when $z = 0$ $\quad \therefore\ 3 - 2\lambda = 0$
$\therefore\ \lambda = \frac{3}{2}$
Then $x = 1 - \frac{3}{2} = -\frac{1}{2}$ and $y = 3 + \frac{3}{2} = \frac{9}{2}$, so the point is $\left(-\frac{1}{2}, \frac{9}{2}, 0\right)$.

b The line meets the YOZ plane when $x = 0$ $\quad \therefore\ 1 - \lambda = 0$
$\therefore\ \lambda = 1$
Then $y = 3 + 1 = 4$ and $z = 3 - 2 = 1$, so the point is $(0, 4, 1)$.

c The line meets the XOZ plane when $y = 0$ $\quad \therefore\ 3 + \lambda = 0$
$\therefore\ \lambda = -3$
Then $x = 1 - (-3) = 4$ and $z = 3 - 2(-3) = 9$, so the point is $(4, 0, 9)$.

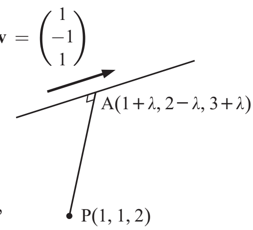
5 Given a line with equations $x = 2 - \lambda,\ \ y = 3 + 2\lambda$ and $z = 1 + \lambda$,
the distance to the point $(1, 0, -2)$ is $\sqrt{(2 - \lambda - 1)^2 + (3 + 2\lambda - 0)^2 + (1 + \lambda + 2)^2}$.

But this distance $= 5\sqrt{3}$ units

$$\begin{aligned}
\therefore\ \sqrt{(1-\lambda)^2 + (3+2\lambda)^2 + (\lambda+3)^2} &= 5\sqrt{3} \\
\therefore\ (1-\lambda)^2 + (3+2\lambda)^2 + (\lambda+3)^2 &= 75 \\
\therefore\ 1 - 2\lambda + \lambda^2 + 9 + 12\lambda + 4\lambda^2 + \lambda^2 + 6\lambda + 9 &= 75 \\
\therefore\ 6\lambda^2 + 16\lambda - 56 &= 0 \\
\therefore\ 3\lambda^2 + 8\lambda - 28 &= 0 \\
\therefore\ (3\lambda + 14)(\lambda - 2) &= 0 \\
\therefore\ \lambda = -\tfrac{14}{3} \ \text{ or } \ \lambda &= 2
\end{aligned}$$

When $\lambda = 2$ the point is $(0, 7, 3)$, and when $\lambda = -\frac{14}{3}$ the point is $\left(\frac{20}{3}, -\frac{19}{3}, -\frac{11}{3}\right)$.

6 **a** Let $A(1+\lambda,\ 2-\lambda,\ 3+\lambda)$ be a point on the line such that $\overrightarrow{PA}$ is perpendicular to the line.



Then $\overrightarrow{PA} = \begin{pmatrix} 1+\lambda-1 \\ 2-\lambda-1 \\ 3+\lambda-2 \end{pmatrix} = \begin{pmatrix} \lambda \\ 1-\lambda \\ 1+\lambda \end{pmatrix}$

and $\text{PA} = \sqrt{\lambda^2+(1-\lambda)^2+(1+\lambda)^2}$
$= \sqrt{\lambda^2+(1-2\lambda+\lambda^2)+(1+2\lambda+\lambda^2)}$
$= \sqrt{3\lambda^2+2}$ units

[PA] is perpendicular to the line when $\text{PA}^2 = 3\lambda^2+2$ is minimised,

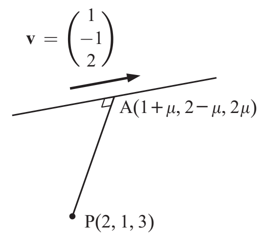
which occurs when $\lambda = -\dfrac{b}{2a} = -\dfrac{0}{6} = 0$

$\therefore$ A is at $(1+0,\ 2-0,\ 3+0)$

$\therefore$ the foot of the perpendicular is $(1,\ 2,\ 3)$.

b Let A be a point on the line such that $\overrightarrow{PA}$ is perpendicular to the line.

$\therefore$ A is at $(1+\mu,\ 2-\mu,\ 2\mu)$ for some μ.



Now $\overrightarrow{PA} = \begin{pmatrix} 1+\mu-2 \\ 2-\mu-1 \\ 2\mu-3 \end{pmatrix} = \begin{pmatrix} \mu-1 \\ 1-\mu \\ 2\mu-3 \end{pmatrix}$

and $\text{PA} = \sqrt{(\mu-1)^2+(1-\mu)^2+(2\mu-3)^2}$
$= \sqrt{\mu^2-2\mu+1+1-2\mu+\mu^2+4\mu^2-12\mu+9}$
$= \sqrt{6\mu^2-16\mu+11}$ units

[PA] is perpendicular to the line when $\text{PA}^2 = 6\mu^2-16\mu+11$ is minimised,

which occurs when $\mu = -\dfrac{b}{2a} = -\dfrac{-16}{12} = \frac{4}{3}$

$\therefore$ A is at $\left(1+\frac{4}{3},\ 2-\frac{4}{3},\ 2(\frac{4}{3})\right)$

$\therefore$ the foot of the perpendicular is $\left(\frac{7}{3},\ \frac{2}{3},\ \frac{8}{3}\right)$.

EXERCISE 16A.3

1 l_1 has direction vector $\binom{4}{-3}$ and l_2 has direction vector $\binom{5}{4}$. If θ is the angle between them,

$$\cos\theta = \frac{\left|\binom{4}{-3} \bullet \binom{5}{4}\right|}{\sqrt{16+9}\sqrt{25+16}} = \frac{|20+(-12)|}{\sqrt{25\times 41}} = \frac{8}{\sqrt{25\times 41}}$$

$\therefore \quad \theta = \arccos\left(\dfrac{8}{\sqrt{25\times 41}}\right) \approx 75.5^o$

$\therefore$ the required angle measures 75.5^o.

2 l_1 has direction vector $\binom{12}{5}$ and l_2 has direction vector $\binom{3}{-4}$. If θ is the angle between them,

$$\cos\theta = \frac{\left|\binom{12}{5} \bullet \binom{3}{-4}\right|}{\sqrt{144+25}\sqrt{9+16}} = \frac{|36+(-20)|}{13\times 5} = \frac{16}{65}$$

$\therefore \quad \theta = \arccos\left(\frac{16}{65}\right) \approx 75.7^o$

3 Line 1 has direction vector $\binom{5}{-2}$ and line 2 has direction vector $\binom{4}{10}$

and $\binom{5}{-2} \bullet \binom{4}{10} = 20+(-20) = 0$

$\therefore$ l_1 and l_2 are perpendicular

4 If $\dfrac{x-8}{3}=\dfrac{9-y}{16}=\dfrac{z-10}{7}=\lambda$ say, then $x=8+3\lambda,\ y=9-16\lambda,\ z=10+7\lambda$

$\therefore$ line 1 has vector $\begin{pmatrix}3\\-16\\7\end{pmatrix}$ and line 2 has vector $\begin{pmatrix}3\\8\\-5\end{pmatrix}$. If θ is the angle between them,

$$\cos\theta=\frac{\left|\begin{pmatrix}3\\-16\\7\end{pmatrix}\bullet\begin{pmatrix}3\\8\\-5\end{pmatrix}\right|}{\sqrt{9+256+49}\sqrt{9+64+25}}=\frac{|9-128-35|}{\sqrt{314}\sqrt{98}}=\frac{154}{\sqrt{314\times 98}}$$

$\therefore\ \theta\approx 28.6^o$

EXERCISE 16B.1

1 **a** **i** When $t=0$, $\binom{x}{y}=\binom{-4}{3}$ $\therefore$ the object is at $(-4, 3)$.

ii The velocity vector is $\binom{12}{5}$.

iii The speed is $\sqrt{12^2+5^2}$ $=13\ \text{m s}^{-1}$

b **i** When $t=0$, $\binom{x}{y}=\binom{0}{-6}$ $\therefore$ the object is at $(0, -6)$.

ii The velocity vector is $\binom{3}{-4}$.

iii The speed is $\sqrt{3^2+(-4)^2}$ $=5\ \text{m s}^{-1}$

c **i** When $t=0$, $\binom{x}{y}=\binom{-2}{-7}$ $\therefore$ the object is at $(-2, -7)$.

ii The velocity vector is $\binom{-6}{-4}$.

iii The speed is $\sqrt{(-6)^2+(-4)^2}$ $=\sqrt{36+16}$ $=\sqrt{52}\ \text{m s}^{-1}$

2 **a** $\binom{4}{-3}$ has length $\sqrt{4^2+(-3)^2}=5$

$\therefore$ $30\binom{4}{-3}$ has length 150

$\therefore$ the velocity vector is $\binom{120}{-90}$.

b $\binom{24}{7}$ has length $\sqrt{24^2+7^2}=25$

$\therefore$ $\frac{1}{2}\binom{24}{7}$ has length 12.5

$\therefore$ the velocity vector is $\binom{12}{3.5}$.

c $2\mathbf{i}+\mathbf{j}=\binom{2}{1}$ has length $\sqrt{2^2+1^2}=\sqrt{5}$

$\therefore$ $10\sqrt{5}\binom{2}{1}$ has length 50

$\therefore$ the velocity vector is $\binom{20\sqrt{5}}{10\sqrt{5}}$.

d $-3\mathbf{i}+4\mathbf{j}=\binom{-3}{4}$ has length $\sqrt{(-3)^2+4^2}=5$

$\therefore$ $20\binom{-3}{4}$ has length 100

$\therefore$ the velocity vector is $\binom{-60}{80}$.

3 Yacht A: $\binom{x_A}{y_A}=\binom{4}{5}+t\binom{1}{-2}$ Yacht B: $\binom{x_B}{y_B}=\binom{1}{-8}+t\binom{2}{1}$, $t\geqslant 0$

a When $t=0$, $\binom{x_A}{y_A}=\binom{4}{5}$ $\therefore$ A is at (4, 5)

and $\binom{x_B}{y_B}=\binom{1}{-8}$ $\therefore$ B is at $(1, -8)$.

b For A, the velocity vector is $\binom{1}{-2}$, and for B it is $\binom{2}{1}$.

c Speed of A $=\sqrt{1^2+(-2)^2}=\sqrt{5}\ \text{km h}^{-1}$. Speed of B $=\sqrt{2^2+1^2}=\sqrt{5}\ \text{km h}^{-1}$.

d The distance between them is
$$\begin{aligned}D&=\sqrt{[(1+2t)-(4+t)]^2+[(-8+t)-(5-2t)]^2}\\&=\sqrt{(-3+t)^2+(-13+3t)^2}\\&=\sqrt{9-6t+t^2+169-78t+9t^2}\\&=\sqrt{10t^2-84t+178}\end{aligned}$$

This is a minimum when $10t^2-84t+178$ is a minimum. This occurs when

$t=\dfrac{-b}{2a}=\dfrac{84}{20}=4.2$ hours. $\therefore$ the time is 4 h 12 min after 6 am, or 10:12 am.

e A has direction vector $\binom{1}{-2}$ and B has direction vector $\binom{2}{1}$.

Since $\binom{1}{-2}\bullet\binom{2}{1}=2-2=0$, the paths of the yachts are at right angles to each other.

4 **a** P has position $\begin{pmatrix} x_1 \\ y_1 \end{pmatrix} = \begin{pmatrix} -5 \\ 4 \end{pmatrix} + t\begin{pmatrix} 3 \\ -1 \end{pmatrix}$ and at $t = 0$, the time is 1:34 pm

$\therefore\ x_1(t) = -5 + 3t, \quad y_1(t) = 4 - t.$

b Speed $= \sqrt{3^2 + (-1)^2} = \sqrt{10}$ km min^{-1}

c Q fires its torpedo after a minutes.

$\therefore$ at time t, its torpedo has travelled for $(t - a)$ minutes.

$\therefore\ \begin{pmatrix} x_2 \\ y_2 \end{pmatrix} = \begin{pmatrix} 15 \\ 7 \end{pmatrix} + (t - a)\begin{pmatrix} -4 \\ -3 \end{pmatrix}, \quad t > a$

$\therefore\ x_2(t) = 15 - 4(t - a)$ and $y_2(t) = 7 - 3(t - a)$

d They meet when $x_1(t) = x_2(t)$ and $y_1(t) = y_2(t)$

$\therefore\ -5 + 3t = 15 - 4(t - a)$ and $4 - t = 7 - 3(t - a)$

$\therefore\ 7t - 4a = 20$ ..... (1) and $2t - 3a = 3$ ..... (2)

Solving simultaneously,

$$21t - 12a = 60 \qquad 3 \times (1)$$
$$-8t + 12a = -12 \qquad (-4) \times (2)$$

adding $13t = 48$

$\therefore\ t = \frac{48}{13}$ and $7\left(\frac{48}{13}\right) - 4a = 20$

$\therefore\ t \approx 3.6923$ $\qquad \therefore\ 5.8462 = 4a$

$\therefore\ t \approx 3$ min 41.54 sec $\qquad \therefore\ a \approx 1.4615 \approx 1$ min 27.7 sec

So, as $a \approx 1.4615$, Q fired at 1:35:28 pm, and the explosion occurred at 1:37:42 pm.

5 **a** $|\mathbf{b}| = \sqrt{(-3)^2 + (-1)^2} = \sqrt{10}$

As the speed is $40\sqrt{10}$ km h^{-1}, the velocity vector is $40\begin{pmatrix} -3 \\ -1 \end{pmatrix} = \begin{pmatrix} -120 \\ -40 \end{pmatrix}$.

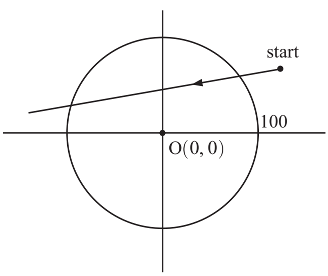
b $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 200 \\ 100 \end{pmatrix} + t\begin{pmatrix} -120 \\ -40 \end{pmatrix}, \quad t \geqslant 0 \quad \{t = 0$ at 12:00 noon$\}$

c At 1:00 pm, $t = 1$ and $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 200-120 \\ 100-40 \end{pmatrix} = \begin{pmatrix} 80 \\ 60 \end{pmatrix}$

d The distance from O(0, 0) to $P_1(80, 60)$ is $\left|\begin{pmatrix} 80 \\ 60 \end{pmatrix}\right| = \sqrt{80^2 + 60^2} = 100$ km,

which is when it becomes visible to radar. {within 100 km of O(0, 0)}

e



A general point on the path is $P(200 - 120t, 100 - 40t)$.

Now $\overrightarrow{OP} = \begin{pmatrix} 200-120t \\ 100-40t \end{pmatrix}$,

and for the closest point $\overrightarrow{OP} \bullet \begin{pmatrix} -3 \\ -1 \end{pmatrix} = 0$

$\therefore\ -3(200 - 120t) - 1(100 - 40t) = 0$

$\therefore\ -700 + 400t = 0$

$\therefore\ t = \frac{7}{4} = 1\frac{3}{4}$ hours

The time when the aircraft is closest is 1:45 pm, and

at this time $\overrightarrow{OP} = \begin{pmatrix} 200-120(\frac{7}{4}) \\ 100-40(\frac{7}{4}) \end{pmatrix} = \begin{pmatrix} -10 \\ 30 \end{pmatrix}$

$\therefore\ d_{\min} = \sqrt{(-10)^2 + 30^2} \approx 31.6$ km

f It disappears from radar when $|\overrightarrow{OP}| = 100$ and $t > 1\frac{3}{4}$

$$\therefore\ \sqrt{(200 - 120t)^2 + (100 - 40t)^2} = 100$$
$$\therefore\ 40\,000 - 48\,000t + 14\,400t^2 + 10\,000 - 8000t + 1600t^2 = 10\,000$$
$$\therefore\ 16\,000t^2 - 56\,000t + 40\,000 = 0$$
$$\therefore\ 16t^2 - 56\,t + 40 = 0 \qquad \{\div 1000\}$$
$$\therefore\ 2t^2 - 7t + 5 = 0 \qquad \{\div 8\}$$
$$\therefore\ (2t - 5)(t - 1) = 0$$
$$\therefore\ t = \tfrac{5}{2} \qquad \{\text{as } t > 1\tfrac{3}{4}\}$$

So, the aircraft disappears from the radar screen $2\frac{1}{2}$ hours after noon, or at 2:30 pm.

6 For A, $x_A(t) = 3 - t$, $y_A(t) = 2t - 4$ For B, $x_B(t) = 4 - 3t$, $y_B(t) = 3 - 2t$

a When $t = 0$, $x_A(0) = 3$, $y_A(0) = -4$ and $x_B(0) = 4$, $y_B(0) = 3$

$\therefore$ A is at (3, −4). $\therefore$ B is at (4, 3).

b The velocity vector of A is $\binom{-1}{2}$ and the velocity vector of B is $\binom{-3}{-2}$.

c If the angle is θ, $\binom{-1}{2} \bullet \binom{-3}{-2} = \sqrt{1+4}\sqrt{9+4}\cos\theta$

$\therefore \quad 3 - 4 = \sqrt{5}\sqrt{13}\cos\theta$

$\therefore \quad \frac{-1}{\sqrt{65}} = \cos\theta$ and so $\theta \approx 97.1^o$

d If D is the distance between them, then

$D = \sqrt{[(4-3t)-(3-t)]^2 + [(3-2t)-(2t-4)]^2}$

$= \sqrt{[1-2t]^2 + [7-4t]^2}$

$= \sqrt{1 - 4t + 4t^2 + 49 - 56t + 16t^2}$

$= \sqrt{20t^2 - 60t + 50}$

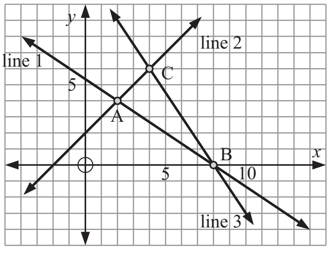
and D is a minimum when

$t = -\dfrac{b}{2a} = \frac{60}{40} = 1\frac{1}{2}$

$\therefore \quad t = 1.5$ hours

EXERCISE 16B.2

1 **a**



b A is (2, 4), B is (8, 0), C is (4, 6)

c $BC = \sqrt{(8-4)^2 + (0-6)^2} = \sqrt{16+36} = \sqrt{52}$ units

$AB = \sqrt{(8-2)^2 + (0-4)^2} = \sqrt{36+16} = \sqrt{52}$ units

$\therefore$ BC = AB and so ΔABC is isosceles.

d Line 1 and Line 2 meet at A.

$\therefore \quad \binom{-1}{6} + \binom{3r}{-2r} = \binom{0}{2} + s\binom{1}{1}$

$\therefore \quad \binom{3r-s}{-2r-s} = \binom{1}{-4}$

$\therefore \quad 3r - s = 1$

and $\quad 2r + s = 4$

Adding, $\quad 5r \quad = 5 \quad \therefore \quad r = 1$

$\therefore \quad \binom{x}{y} = \binom{-1}{6} + \binom{3}{-2} = \binom{2}{4}$ ✓

Line 2 and Line 3 meet at C.

$\therefore \quad \binom{0}{2} + s\binom{1}{1} = \binom{10}{-3} + t\binom{-2}{3}$

$\therefore \quad \binom{s+2t}{s-3t} = \binom{10}{-5}$

$\therefore \quad s + 2t = 10$

$\quad -s + 3t = 5$

Adding, $\quad 5t = 15 \quad \therefore \quad t = 3$

$\therefore \quad \binom{x}{y} = \binom{10}{-3} + 3\binom{-2}{3} = \binom{4}{6}$ ✓

Line 1 and Line 3 meet at B.

$\therefore \quad \binom{-1}{6} + r\binom{3}{-2} = \binom{10}{-3} + t\binom{-2}{3}$

$\therefore \quad \binom{3r+2t}{-2r-3t} = \binom{11}{-9}$

$\therefore \quad 3r + 2t = 11$ ..... (1)

$\quad -2r - 3t = -9$ ..... (2)

$\therefore \quad 9r + 6t = 33$ $\{3 \times (1)\}$

$\quad -4r - 6t = -18$ $\{2 \times (2)\}$

Adding, $\quad 5r \quad = 15$

$\therefore \quad r = 3$

So, $\binom{x}{y} = \binom{-1}{6} + 3\binom{3}{-2} = \binom{8}{0}$ ✓