MATHEMATICS 10 MYP 5 (Extended) third edition

Chapter summaries

Haese Mathematics

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ASSUMED KNOWLEDGE: LINEAR EQUATIONS

A Linear equations
B Problem solving with equations

Keywords:

- algebraic equation
- equal sign
- equation
- inverse operations
- left hand side
- linear equation
- right hand side

Since the students will not have done as much linear equations in earlier years as they had in the previous edition, we felt it was sensible to include this online chapter for students who are still struggling with linear equations.

The chapter does contain some algebraic expansion, which is presented in Chapter 2 of the textbook, but the only expansion used is the distributive law, which students will have seen in MYP 3 and MYP 4.

Placing this chapter in Assumed Knowledge, instead of together with linear inequalities as was done in MYP 4, allows us to place Sets and Venn diagrams together in a single chapter. It also means that all of the work involving inequalities (linear inequalities, graphing inequalities on the Cartesian plane, and linear programming) can be studied together, rather than being broken across chapters.

CHAPTER 1: EXPONENTS

A Exponent laws
B Rational exponents
C Standard form (scientific notation)

Keywords:

- base
- exponent
- exponent laws
- exponent notation
- index
- power
- rational exponent
- scientific notation
- standard form

This chapter has been renamed from "Indices" to match the terminology used in the MYP Framework.

The material in Section A should be familiar to students from MYP 3 and MYP 4, so it should be a gentle introduction to the year.

Section B gives students their first look at rational exponents. This may be quite a conceptual leap for many students, since it is not immediately clear what it means to raise a number to a fractional power. Students should be reminded that they have already encountered such a leap when defining the zero and negative exponents in previous years – what we are doing is giving a meaningful interpretation of these exponents such that the existing exponent laws still hold true. In the first Investigation, we establish that the only meaningful interpretation of $a^{\frac{1}{n}}$ is the nth root of a.

In Question **4** of Exercise 1B.1, students write algebraic expressions involving roots as powers of x. This will be useful when studying calculus in later years.

In Section C, we have adjusted our approach to explaining standard form, to give students more guidance in how to choose their values of a and k in $a \times 10^k$. This section extends what was done in MYP 4 by also considering the addition and subtraction of numbers in standard form. Students should write the terms with the same power of 10 to perform the addition or subtraction. Through practice, students should see that it is better to adjust the smaller value so that its power of 10 matches that of the larger value. This is because the power of 10 of the larger value is more likely to be the correct power of 10 required to write the final answer in standard form.

CHAPTER 2: ALGEBRA: EXPANSION AND FACTORISATION

A The distributive law
B The product $(a+b)(c+d)$
C Difference between two squares expansion
D Perfect squares expansion
E Further expansion
F The binomial expansion
G Factorisation
H Difference between two squares factorisation
I Perfect squares factorisation
J Expressions with four terms
K Factorising x^2+bx+c
L Factorising $ax^2+bx+c,\ a \neq 1$
M Miscellaneous factorisation

Keywords:

- binomial
- binomial expansion
- difference between two squares
- distributive law
- expansion
- factorisation
- fully factorised
- linear factors
- perfect squares
- quadratic trinomial
- splitting the middle term
- sum and product method

In this edition, we have given a more complete coverage of the expansion laws, rather than just presenting a single section of "Revision of the expansion laws". Given that students now will have only seen most of this material for the first time in MYP 4, it seems reasonable to spend a little more time at MYP 5 to reinforce this work.

In Section C, some of the expansions involve working with surds. Although this comes before Chapter 4 (Surds and other radicals), it only involves very intuitive calculations such as $(\sqrt{2})^2 = 2$ based on the definition of a surd. Students should be familiar with these sorts of calculations from previous years.

In Section E (Further expansion), it may help students to recognise that they have already encountered the idea of "multiplying each term in the first bracket by each term in the second bracket", since that is what happens when applying the FOIL rule to $(a+b)(c+d)$ in Section B.

Section F (The binomial expansion) will be new to students. The Investigation on Pascal's triangle at the end of the section gives students a hint towards generating a general binomial expansion, which students may explore further in their Diploma Programme course.

Students should be familiar with the factorisation material in Sections G to I from MYP 4, so these sections can be worked through quickly if needed.

Section J (Expressions with four terms) is primarily a lead-in to factorising by "splitting" the middle term in Section L. It should be made clear to students that most expressions with four terms cannot be factorised in this way.

In the Discussion in Section K, students should find that:

- If the sum and product of two numbers are both positive, the numbers must both be positive.
- If the sum and product of two numbers are both negative, the numbers are opposite in sign, and the negative number has the largest absolute value.
- If the sum is positive and the product is negative, the numbers are opposite in sign, and the positive number has the largest absolute value.
- If the sum is negative and the product is positive, the numbers must both be negative.

In Section L, students will study factorisation by "splitting" the middle term. Some students may have studied this in MYP 4, but others will be seeing it for the first time here. It may help students to see that this is a more general approach to the method used in Section K, since when $a = 1$ the method is essentially reduced to the sum and product method. The only difference is that the factorisation must be completed using the technique studied in Section J. Students should convince themselves that the order in which they write the split terms does not matter, since the resulting factorisation will be the same in either case.

In Section M, students will need to choose which factorisation method to use. Students may wish to produce their own summaries describing when each method is suitable.

CHAPTER 3: SETS AND VENN DIAGRAMS

A Sets
B Complement of a set
C Intersection and union
D Special number sets
E Interval notation
F Venn diagrams
G Venn diagram regions
H Numbers in regions
I Problem solving with Venn diagrams

Keywords:

- complement
- complementary sets
- disjoint
- element
- empty set
- equal sets
- finite set
- infinite set
- integers
- intersection
- interval notation
- irrational numbers
- member
- mutually exclusive
- natural numbers
- negative integers
- positive integers
- rational numbers
- real numbers
- set
- set identity
- subset
- union
- universal set
- Venn diagram

Although Sets and Venn diagrams were presented in separate chapters in MYP 4, they have been placed in a single chapter here, since we are now assuming they have the linear equation knowledge to find unknown numbers of elements in Venn diagrams.

When discussing the union "A or B", it is important to emphasise that elements in both A and B are included in the union. This is a good opportunity to discuss how words can be used differently in mathematics than they are in everyday use, as "or" is often used to mean "one or the other, but not both" in everyday use.

We have endeavoured to extend what was done in the MYP 4 sets chapter by giving more opportunities to explore the concepts of finite and infinite sets. In the Discussion at the end of Section D, students should consider that, when we have two *finite* sets A and B where A is a subset of B, it is clear that there are more elements in B than in A. However, it is less clear when A and B are both infinite sets! It is tempting to say that there are about twice as many elements in $\mathbb{Z}$ as in $\mathbb{Z}^+$, since $\mathbb{Z}$ contains all the elements of $\mathbb{Z}^+$, as well as the corresponding negative integers. But does it even make sense to say that one infinite set has *more* elements than another infinite set? A potentially more illuminating example may be if A is the set of even integers, and B is the set of positive integers. Clearly A is a subset of B, but one could easily generate each element of A by multiplying each element of B by 2! Does this mean that, in some sense, they have the same number of elements?

A similar question occurs in the second dot point. Since the interval of numbers from 0 to 1 appears smaller than the interval of numbers greater than 1, it seems logical that there are more numbers greater than 1 than there are numbers between 0 and 1. However, for each number greater than 1, there is a corresponding number between 0 and 1, which is found by taking the reciprocal of the original number. Does this correspondence mean that there are the same number of real numbers between 0 and 1 as there are greater than 1?

When dealing with these questions, students should be reminded that rules that apply to finite sets do not necessarily extend to infinite sets.

The Venn diagram material at MYP 5 is extended from what was done in MYP 4, to consider more Venn diagrams with three sets, and to prove some set identities.

Section 2H (The algebra of sets) in the previous edition has been converted into an Activity at the end of Section G.

In the Puzzle at the end of Section G, students should be encouraged to place numbers in the Venn diagram, starting with 1, 2, 3, ... and so on, until each region contains at least one element. The smallest element in each region then forms the universal set.

When problem solving with three set Venn diagrams in Section I, students should be aware that the difficulty of the problems depends very much on the combination of information given. For example, Worked Example 10 shows the most difficult case, in which the number of elements in all three sets is unknown. This means that all of the known information must be expressed in terms of this unknown value x. However, Questions **9** and **10** immediately following are the easiest case, in which the number of elements in all three sets is known, and the rest of the regions can be deduced quite easily from the remaining information. Question **11** is slightly harder, and Question **12** is the most difficult form outlined in the worked example.

CHAPTER 4: SURDS AND OTHER RADICALS

- **A** Radicals
- **B** Properties of radicals
- **C** Simplest surd form
- **D** Power equations
- **E** Operations with radicals
- **F** Division with surds
- **G** Equality of surds

Keywords:

- cube root
- equality of surds theorem
- integer denominator
- nth root
- power equation
- radical
- radical conjugate
- rationalising the denominator
- simplest surd form
- square root
- surd

In Section A, students perform calculations based purely on the definition of the square root, such as $\sqrt{5} \times \sqrt{5} = 5$. In Section B, students use properties of square roots, such as $\sqrt{a} \times \sqrt{b} = \sqrt{ab}$, to perform calculations.

In Section C, more able students may be asked to consider situations where simplest surd form is useful, and situations where it is better to leave the surd in its original form. For example, when comparing which value is larger, it is easier to compare $\sqrt{27}$ and $\sqrt{28}$ than to compare $3\sqrt{3}$ and $2\sqrt{7}$.

In this edition we have added Section D (Power equations). We feel that it is important to consider power equations, as they are useful for solving problems in topics such as measurement, proportion, and similarity, but are rarely addressed in their own right.

In the Discussion at the end of Section D, students should find that, when both sides of an equation are squared, we often introduce an extra solution. There are often good reasons to square both sides of an equation, but we need to be aware that this may create extra solutions, and check whether the solutions we obtain all satisfy the original equation.

The sections "Adding and subtracting radicals" and "Multiplications involving radicals" in the previous edition have been combined into the single section "Operations with radicals". Since the rules for operating with radicals are identical to those for algebra, we felt there was little to be gained by keeping these sections separate.

In Section F, students are likely to have performed divisions such as $\frac{5}{\sqrt{2}}$ in MYP 4, however they will not have seen divisions such as $\frac{1}{2+\sqrt{2}}$. In this edition, we give a more general definition of "radical conjugate", which means we do not need to go through all the cases individually in the exercise.

In Section G, we give a more detailed explanantion of proof by contradiction, which is used to establish the irrationality of $\sqrt{2}$. This is likely to be useful for students moving on to the Mathematics: Analysis and Approaches HL course in the Diploma Programme.

CHAPTER 5: PYTHAGORAS' THEOREM

A Pythagoras' theorem
B Pythagorean triples
C Problem solving
D The converse of Pythagoras' theorem

Keywords:

- converse of Pythagoras' theorem
- Pythagoras' theorem
- Pythagorean triple

In this edition, we have changed the order of the sections, so that the converse of Pythagoras' theorem is done at the end. This allows us to consider the non-contextual and contextual applications of Pythagoras' theorem first, before considering the converse.

The section on circle problems has been removed in this edition, and these problems have been moved to the appropriate sections of Chapter 14 (Circle geometry). This approach means that we do not need to present the relevant circle theorems twice.

The questions involving three-dimensional objects have been absorbed into Section C (Problem solving), rather than being presented as a section of their own. We feel that presenting them in a section of their own makes them seem more difficult than they really are.

CHAPTER 6: ALGEBRAIC FRACTIONS

A Evaluating algebraic fractions
B Simplifying algebraic fractions
C Multiplying algebraic fractions
D Dividing algebraic fractions
E Adding and subtracting algebraic fractions
F Equations with algebraic fractions

Keywords:

- algebraic fraction
- equating numerators
- evaluate
- lowest common denominator
- lowest terms
- rational expression
- reciprocal

The structure of this chapter remains largely unchanged from what occurred in the previous edition. The main difference is that multiplication and division of algebraic fractions have been split into separate sections.

Placing this chapter after we have studied factorisation allows students to factorise the numerator and denominator of an algebraic fraction. This helps in cancelling common factors when simplifying, multiplying, or dividing algebraic fractions.

In Section E, we have avoided using the term "simplify" as much as possible, as this may be ambiguous. For example, in some questions students must write the sum of two fractions as a single fraction, whereas in other questions students must take a single fraction such as $\frac{x+9}{3}$, and write it as the sum of two parts. It is therefore unclear which form is the "simplest" in this case. Instead we have been more explicit about what the student should do in each question.

This may be a good opportunity to discuss the merits of the term "simplify", and to help students understand that when we manipulate an algebraic expression, we are turning it into a different form. Whether this new form is "better" or "simpler" may depend on what we are trying to do with the expression.

In Section F, we have greatly expanded the types of equations involving algebraic fractions which can be solved, in particular to make use of the addition and subtraction of algebraic fractions done in Section E. We have also included some problem solving questions.

CHAPTER 7: FORMULAE

A Formula construction
B Substituting into formulae
C Rearranging formulae
D Rearrangement and substitution

E Predicting formulae

Keywords:

- formula
- inverse operations
- rearrange
- subject
- substitution

In this edition, we have added a Measurement chapter to this book. As a result, we have removed any questions in this chapter which refer to measurement formulae, as these will be addressed in the measurement chapter. In their place, we have included more questions based on physics formulae.

We have also included more questions in which students must solve a power equation to find an unknown variable. The work done in Chapter 4 should help with this.

In Section D, students are asked to rearrange the formula to make a particular variable the subject, and then use substitution to evaluate that variable in different circumstances. The students are always asked to evaluate the variable multiple times, as this serves to highlight the advantage of rearranging the formula first. If the formula is rearranged first, the rearrangement only needs to be done once, rather than for each time the variable must be evaluated.

In Section E, students observe the first few numbers in a sequence, identify the pattern in the sequence, and use this information to generate a formula for the nth term.

CHAPTER 8: MEASUREMENT

A Length and perimeter
B Area
C Surface area
D Volume
E Capacity

Keywords:

- arc length
- area
- capacity
- centimetre
- circumference
- hectare
- kilometre
- length
- metre
- millimetre
- perimeter
- surface area
- tapered solid
- volume

We have added this chapter in this edition, as measurement is now a part of all of the Diploma Programme courses, including the HL courses.

Placing this chapter after the Formulae chapter allows us to include questions in which students must find an unknown which is not the subject of the formula, for example, finding the radius of a circle given its circumference.

In this edition, all of the measurement formulae given here, including the surface area and volumes of tapered solids and spheres, were introduced in MYP 4. This means that in MYP 5 there is less need to derive each of these formulae, as this was largely done in MYP 4, where appropriate.

It is important that classes move through this content quickly if students are comfortable with the material, as there is a lot of other more rigorous content which must be covered in MYP 5.

In order to provide some more challenging measurement material, we have included an Investigation at the end of Section C, in which students are led through Archimedes' derivation of the surface area of a sphere by dividing it into smaller and smaller pieces. A lot of the ideas explored here form the precursor to the development of calculus.

CHAPTER 9: QUADRATIC EQUATIONS

A Equations of the form $x^2 = k$
B The null factor law
C Solving by factorisation
D Completing the square
E The quadratic formula

F Problem solving
G Quadratic equations with $\Delta < 0$
H The sum and product of the roots

Keywords:

- completed square form
- completing the square
- complex number
- discriminant
- imaginary number
- null factor law
- quadratic equation
- quadratic formula
- real quadratic equation
- root

In Section A (Equations of the form $x^2 = k$), students may notice that simple versions of these equations were solved in Section 4D (Power equations). Here we use the same principle to solve more complicated equations such as $(3x-2)^2 = 10$.

In this edition, we have given the null factor law a section of its own (Section B). We feel this is appropriate seeing as quadratic equations have now been removed from MYP 3, so this is now only the second time students have encountered quadratic equations.

In Section C.2, students should recognise that equations such as $x^2 - 9 = 0$, which can be solved by difference between two squares factorisation, could also be solved by rearranging it to $x^2 = 9$. However, for more complicated equations such as $(2x+1)^2 - (x+2)^2 = 0$, it should be clear that using difference between two squares factorisation is more efficient.

Students may have solved quadratic equations by completing the square in MYP 4. In MYP 5 we extend this work to give a more detailed treatment of completing the square with $a \neq 1$ in Section D.

Students should find that solving quadratic equations by completing the square is quite tedious, and that by applying completing the square to the general quadratic equation $ax^2 + bx + c = 0$, we can obtain the quadratic formula for the solution to the equation in terms of a, b, and c, without having to perform all of the steps each time.

In the Discussion in Section E, students should be able to solve the quadratic equation using factorisation, completing the square, and using the quadratic formula. They should also find that using factorisation is the quickest method in this case. This is a good opportunity to explain that the quadratic formula *can* be used for any quadratic equation, however it is worth checking whether the quadratic equation can be solved by factorisation first, since this method will be much quicker.

In this edition, similarity has been moved later in the book, and now appears after quadratic equations. For this reason, the questions involving similar triangles have been removed from Section F, and now appear in the Congruence and similarity chapter.

Section G (Quadratic equations with $\Delta < 0$) is marked as Extended material, and is recommended for students intending to study one of the HL courses in the Diploma Programme. In this section, we define the imaginary number i, and use it to solve quadratic equations with no real solutions. This introduction to complex numbers will be very useful preparation for either of the HL courses.

Section H (The sum and product of the roots) has been marked in green, and is recommended for students intending to study the Mathematics: Analysis and Approaches HL course in the Diploma Programme. In this section, we look at the sum and product of the roots of quadratic equations. This work is extended in the Mathematics: Analysis and Approaches HL course to consider the sum and product of the roots of polynomials in general.

CHAPTER 10: COORDINATE GEOMETRY

A The distance between two points
B Midpoints
C Gradient
D Parallel and perpendicular lines
E Using coordinate geometry
F The equation of a line
G Graphing straight lines
H Finding the equation of a line
I Perpendicular bisectors
J 3-dimensional coordinate geometry

Keywords:

- axes intercepts
- Cartesian plane
- coordinates
- distance formula
- equation of a line
- general form
- gradient
- gradient formula
- gradient-intercept form
- midpoint
- negative reciprocals
- number plane
- ordered pair
- origin
- parallel lines
- perpendicular bisector
- perpendicular lines
- point-gradient form
- quadrant
- x-axis
- X-axis
- x-coordinate
- x-intercept
- y-axis
- Y-axis
- y-coordinate
- y-intercept
- Z-axis

In this edition, we have moved this chapter later in the book, so that it occurs after quadratic equations. This allows us to use quadratic equations to solve problems involving the distance between points in Section A.

In Section D, we have adjusted how we derive the relationship between the gradients of parallel and perpendicular lines. We first use the converse of Pythagoras' theorem to establish that the gradients of perpendicular lines are negative reciprocals. We then use this fact to show that two parallel lines, which are both cut by the same perpendicular transversal, must have equal gradients.

In the Discussion in Section D, students should find that the rule for gradients of perpendicular lines does not apply to horizontal and vertical lines, since vertical lines have undefined gradient. The idea that vertical and horizontal lines are perpendicular should be intuitive to students, and gradient is not a very useful tool in this case.

Knowledge of the gradient of parallel and perpendicular lines allows us to verify and prove geometric facts in Section E (Using coordinate geometry). Many of the questions in this section were part of Section D in the previous edition.

Section F (The equation of a line) is largely a new section for this edition, in which students convert between different forms of the equation of a line, and decide whether points lie on given lines. The section of the same name in the previous edition was largely concerned with *finding* the equation of a line given particular information. This work now appears in Section H.

Section G (Graphing straight lines) is also a new section for this edition. We felt that graphing a straight line from its equation is an important skill. It will be used when solving simultaneous equations, and in linear programming.

We have restructured the text in Section H so that, rather than focusing on the form in which to write the equation, we consider the different combinations of information students may be given about a line, and how to find the equation of the line in each case.

Section I (Perpendicular bisectors) is a useful application of the work students have been studying. It will also be useful for students planning on completing one of the Mathematics: Applications and Interpretation courses in the Diploma Programme, since Voronoi diagrams, which involve perpendicular bisectors, are part of these courses.

The material which appeared in Section G (Distance from a point to a line) in the previous edition has been removed from this chapter. Since this work requires solving simultaneous equations, and this book now contains a simultaneous equations chapter, this work has been moved to an Investigation in that chapter (Chapter 11).

In Section J (3-dimensional coordinate geometry), students should be reminded that, although it is harder to visualise coordinates in three dimensions, most of the work that is done (such as finding distances and midpoints) extends fairly logically from what they have seen in two dimensions.

CHAPTER 11: SIMULTANEOUS EQUATIONS

A Graphical solution
B Solution by substitution
C Solution by elimination
D Problem solving
E Non-linear simultaneous equations

Keywords:

- elimination
- simultaneous equations
- simultaneous solution
- substitution

In this edition of the MYP books, simultaneous equations appears at MYP 4 and MYP 5, rather than MYP 3 and MYP 4. So, this chapter has been added in this edition, and the chapter that appears here is adapted from the chapter that previously appeared in MYP 4.

When studying simultaneous equations, it is important that students understand the conceptual shift in that our solution takes the form of a value of x *and* y which make both equations true simultaneously.

In Section A, a graphical approach is used. This should allow students to use what they learnt in the previous chapter, and see that by graphing the line corresponding to each equation, the intersection point gives us the solution to the simultaneous equations.

This approach should illustrate to students why some systems have no solutions or infinitely many solutions. However, reading the solution from a graph makes it difficult to find non-integer solutions accurately. This leads to a need for the algebraic approaches outlined in Sections B and C.

Towards the end of Section D (Problem solving), students may find that the equations they are generating do not appear to be linear simultaneous equations. However, some of the questions require the students to do some rearrangement, and other questions require students to recognise that the equations are inherently linear, even if the variables take forms such as x^2 or y^2.

The Investigation about finding the distance from a point to a line at the end of Section D has been adapted from Section 6G in the previous edition. Now that there is a simultaneous equations chapter, it seemed more logical to place this material here.

CHAPTER 12: LINEAR INEQUALITIES

A Linear inequalities
B Problem solving with inequalities
C Regions of the Cartesian plane
D Feasible region or simplex
E Linear programming
F Problem solving with linear programming

Keywords:

- Cartesian plane
- constraint
- double inequality
- feasible region
- feasible solution
- inequality
- linear inequality
- linear programming
- objective
- objective function
- optimal solution
- optimal value
- simplex

This chapter combines the work done in Section 22B (Linear inequalities) and Chapter 29 (Linear programming) of the previous edition.

When solving linear inequalities, students can use the work done in Chapter 3 to write the solutions in interval notation, and to graph the solution on a number line.

In this edition, we include double inequalities, such as $5 < 3x - 4 < 17$. When solving double inequalities, students must remember to write the expressions in the solution from smallest to largest. So, if students have obtained the solution $8 > x > 5$, they must write this as $5 < x < 8$. Students should not think of this as a mathematical step of "reversing the inequality signs" in the same way as when they multiply or divide by a negative number. Instead, it is more about rewriting the solution in the correct format.

In the Discussion at the end of Section A, students should find that double inequalities such as $x + 1 \leqslant 3x - 5 < 10$ can be solved by treating them as two separate inequalities, and then combining the solutions. For example, the solution to $x + 1 \leqslant 3x - 5$ is $x \geqslant 3$, and the solution to $3x - 5 < 10$ is $x < 5$. So, the solution to the double inequality occurs where *both* of these are true, which is the interval $3 \leqslant x < 5$.

The Puzzle at the end of Section B can be solved intuitively or algebraically. Intuitively, students should see that the number of "Yes" responses on Island C will be minimised if we assume all of the migration to the other two islands came from Island C. In this case, 35 people initially on Island C have moved to another island (30 to Island A, 5 to Island B). So, there would be $100 - 35 = 65$ people originally on Island C who are still on Island C.

Also, the number of "Yes" responses on Island C will be maximised if we maximise the amount of migration between the other two islands. If all 5 "No" responses on Island B came from Island A, and 15 "No" responses from Island A came from Island B (the maximum amount possible), then only 15 people initially on Island C moved to another island (to Island A, in this case). So, $100 - 15 = 85$ people originally on Island C would still be on Island C. So, the number of "Yes" responses on Island C could range from 65 to 85 (inclusive).

In Section C (Regions of the Cartesian plane) students graph regions such as $2x + 5y < 10$ by first graphing the line $2x + 5y = 10$, and then using a test point such as $(0, 0)$ to determine which side of the line the required region lies on. In the Discussion at the end of Section C, students should conclude that they can quickly check the coordinates of a vertex of a region by verifying that it satisfies the equations of the lines which define that vertex.

In this edition, we have vastly improved the structure of the linear programming material. In Section D, students learn to convert a real-world problem into a set of constraints, and to graph the region defined by those constraints, known as the simplex.

The Discussion at the start of Section E should lead students to the idea that, while there are likely to be many *feasible* solutions, many of the solutions are not likely to be sensible. In addition, in order to work out the *best* combination, we need to define an objective, such as maximising seating capacity, or minimising cost. The Investigation should help students conclude that the optimal value of a linear expression will always occur at a vertex of the simplex.

In the Discussion at the end of Section E, students should find that there will be more than one solution to a linear programming problem if the objective function is parallel to one of the constraint lines. Problems of this type are explored further in Worked Example 16, and Questions **8** and **9** of Exercise 12F.

CHAPTER 13: CONGRUENCE AND SIMILARITY

A Congruent triangles
B Proof using congruence
C Similar triangles
D Areas and volumes of similar objects

Keywords:

- congruent figures
- congruent triangles
- equiangular
- similar figures
- similar triangles

Although congruence and similarity does appear in our MYP 3 book, it is not explicitly required in the MYP Framework until MYP 4. For this reason, this may be the second or the third time students have seen congruence and similarity. If students are seeing this work for the third time, the material can be worked through more quickly, especially Sections A to C.

Students may find the Investigation at the end of Section B to be quite subtle. For example, in **2**, students should be reminded that they cannot assume that the sides of the triangle are equal, since that is what we are trying to prove. In part **a**, we can only establish that two pairs of sides are equal, and the equal angles are not between the equal sides. So, we cannot conclude that the triangles formed are congruent. In part **b**, we can use AAcorS to prove that the triangles are congruent, and hence the large triangle is isosceles.

The Investigation on the midpoint theorem in the previous edition has been removed, and most of the material has been moved to the Deductive geometry chapter of MYP 4.

In Section C, some questions which involve solving a quadratic equation have been added. These questions were in the quadratic equations chapter in the previous edition, but needed to be moved here because the quadratic equations chapter is now placed before similarity.

Whereas areas and volumes of similar objects were given separate sections in MYP 4, they are considered in one section here. This gives more opportunity to explore the relationships between lengths, surface areas, and volumes of similar three-dimensional objects.

CHAPTER 14: CIRCLE GEOMETRY

A Angle in a semi-circle theorem
B Chords of a circle theorem
C Radius-tangent theorem
D Tangents from an external point theorem
E Angle between a tangent and a chord theorem
F Angle at the centre theorem
G Angles subtended by the same arc theorem
H Cyclic quadrilaterals
I Tests for cyclic quadrilaterals

Keywords:

- chord
- concyclic
- cyclic quadrilateral
- diameter
- inscribed
- radius
- semi-circle
- subtended
- tangent

In this edition, rather than dividing the circle theorems arbitrarily into a "Circle theorems" section and a "Further circle theorems" section, each theorem is given a separate section.

Since the "Circle problems" section of the chapter on Pythagoras' theorem has been removed, some of the questions that were in that section have been added to this chapter.

In this edition, we have removed the "Geometric proof" section. The problems in this section have been moved to the section corresponding to the relevant circle theorem. An advantage with this approach is that students can prove each circle theorem at the time that the theorem is used, rather than leaving all of the circle theorem proofs until the end.

Some of the problems that involved trigonometry have been removed in this edition. This is because the Circle geometry chapter has been moved earlier in the book, and is now placed before trigonometry.

In this edition, we have split "Cyclic quadrilaterals" (Section H) and "Tests for cyclic quadrilaterals" (Section I) into separate sections.

CHAPTER 15: TRIGONOMETRY

A Labelling right angled triangles
B The trigonometric ratios
C Finding side lengths
D Finding angles
E Problem solving
F True bearings

Keywords:

- adjacent side
- angle of depression
- angle of elevation
- cosine
- hypotenuse
- inverse cosine
- inverse sine
- inverse tangent
- opposite side
- sine
- tangent
- trigonometry
- true bearing
- true north

In this edition, we have presented the material on right angled triangle trigonometry and non-right angled triangle trigonometry in separate chapters. This chapter covers the right angled triangle trigonometry material, which corresponds to Sections A to D of Chapter 12 (Trigonometry) in the previous edition.

When introducing the trigonometric ratios in Section B, the aim should be not only to familiarise students with the side lengths involved in each trigonometric ratio, but to help them understand that a ratio such as $\sin 57°$ is not just an abstract term, but an actual number whose value can be determined by measuring sides of right angled triangles.

In Section E (Problem solving), some questions which use circle geometry have been added. This has been done since this chapter now appears after the Circle geometry chapter.

The work on true bearings (Section F) extends what is done in MYP 4 to consider multi-leg journeys, in which students must find the bearing of the end point from the starting point. In this chapter, the bearings of each leg are chosen to create a right angled triangle. This acts as a lead-in to the more realistic and complex problems in the following chapter, where the bearings can take any value, and distances and bearings are found using non-right angled triangle trigonometry.

As with the chapter on Pythagoras' theorem, the questions involving three-dimensional objects have been absorbed into the section on problem solving, rather than being presented as a section of their own.

The material on projections, and the angle between a line and a plane, has been replaced by an Investigation at the end of Section E, which was in the MYP 4 book in the previous edition.

CHAPTER 16: NON-RIGHT ANGLED TRIANGLE TRIGONOMETRY

A Trigonometry with obtuse angles
B The area of a triangle
C The sine rule
D The cosine rule
E Problem solving

Keywords:

- cosine rule
- sine rule
- unit circle

This chapter consists of the material in Sections E to I of Chapter 12 (Trigonometry) in the previous edition.

Non-right angled triangle trigonometry does appear in our MYP 4 book, but is listed as Extended material at MYP 4, so this is likely to be the first time students have seen non-right angled triangle trigonometry.

In Section A we define the trigonometric ratios for obtuse angles. It may be beneficial to students to see that, although it is not meaningful to talk about the trigonometric ratios for obtuse angles based on our original definition involving right angled triangles, we can extend our definition of trigonometric ratios in a way that is consistent with our original definition, but is also meaningful for a greater range of angles. This is similar to what we did in Chapter 1, when we extended the definition of exponents so that it is meaningful for rational exponents.

In Section A we also establish the relationship between the trigonometric ratios of supplementary angles, which will be useful when considering the sine and cosine rules later in the chapter.

In Section B (The area of a triangle), Question **2** should help students see that the standard formula for the area of a triangle $A = \frac{1}{2} \times \text{base} \times \text{height}$, is a special case of the formula given here, in which the included angle is a right angle.

In Section C.2, students use the sine rule to find unknown angles, and will encounter the ambiguous case. It may be useful for students to consider a situation where we would use the sine rule to find an angle, in which we know two sides of a triangle, and a non-included angle. From their work in congruence, students should realise that this is not necessarily enough information to describe a triangle uniquely. This should help them understand why the ambiguous case arises.

In Section D (The cosine rule), students may benefit from seeing that Pythagoras' theorem is a special case of the cosine rule, in which the included angle is a right angle.

In Section E (Problem solving), students were introduced to true bearings in Chapter 15, so no further explanation of them is given here. The questions in this chapter allow a greater range of scenarios to be considered, where the cosine and sine rules are used to find distances and bearings.

CHAPTER 17: PROBABILITY

A Sample space and events
B Theoretical probability
C The addition law of probability
D Independent events
E Dependent events
F Experimental probability
G Expectation
H Conditional probability

Keywords:

- 2-dimensional grid
- certain event
- combined events
- complementary events
- compound events
- conditional probability
- dependent events
- disjoint
- event
- expectation
- experimental probability
- frequency
- impossible event
- independent events
- mutually exclusive
- probability
- relative frequency
- sample space
- theoretical probability
- tree diagram

The contents of this chapter have been restructured to mirror the structure used in MYP 4.

Section A (Sample space and events) considers not only the different ways to represent the sample space of an experiment, but also the definition of an event connected to an experiment, and how the outcomes of a particular event can be highlighted within the sample space. This serves to give this section some more substance, justifying its inclusion at MYP 5.

In Section B (Theoretical probability), the method for finding unknown numbers in Venn diagram regions has been updated to reflect the more formal method described in Chapter 3.

In the Puzzle at the end of Section B, students should be able to recognise that an easy way to solve the puzzle is to add 1 to each face of one die (so it is numbered 2 to 7), and subtract 1 from each face of the other die (so it is numbered 0 to 5). However, students who find this solution should be challenged to find a less trivial solution; one that involves the same number appearing on a die more than once.

Section C (The addition law of probability) is effectively a restructure of Section G (Mutually exclusive and independent events) from the previous edition. Rather than first considering mutually exclusive events and then generalising to the addition law, here we focus on the addition law of probability, and then observe how the law changes in the special case that A and B are mutually exclusive events. In this case $\mathrm{P}(A \cap B) = 0$, and so $\mathrm{P}(A \cup B) = \mathrm{P}(A) + \mathrm{P}(B)$. The remaining theory about independent events was removed, and some relevant questions were moved to Section D.

At the end of Section H, we have added an Investigation about the conditional probability formula. Students should recognise that the formula as shown in **1 a** is just a formalisation of what they were calculating when finding conditional probabilities from Venn diagrams, two-way tables, and two-dimensional grids in Exercise 17H. Students should also notice that the form of the formula given in **1 b** is preferable in situations where we are dealing with theoretical probabilities, rather than numbers of elements. Questions **2** to **4** provide some practice in using this formula.

CHAPTER 18: STATISTICS

A Discrete numerical data
B Continuous numerical data
C Describing the distribution of data
D Measures of centre
E Box plots
F Cumulative frequency graphs
G Standard deviation
H The normal distribution

Keywords:

- bimodal
- bimodal distribution
- box plot
- categorical data
- categorical variable
- census
- class interval
- column graph
- continuous numerical variable
- cumulative frequency
- cumulative frequency graph
- discrete numerical variable
- distribution
- dot plot
- five-number summary
- frequency histogram
- histogram
- interquartile range
- interval midpoint
- lower quartile
- maximum value
- mean
- median
- minimum value

- modal class
- mode
- negatively skewed distribution
- normal distribution
- numerical data
- numerical variable
- outlier
- parallel box plot
- percentile
- population
- positively skewed distribution
- range
- sample
- standard deviation
- statistics
- survey
- symmetric distribution
- tally and frequency table
- upper quartile
- variable

This chapter has been moved much later in the book so that it is grouped together with probability and bivariate statistics.

The chapter opens in much the same way as the previous edition. However, we have moved the material on "Describing the distribution of data" to its own section to match what we did for MYP 4.

The "Demographics" Global Context on page 381 is a good opportunity to practise skills from both probability and statistics in the context of data for human populations.

Cumulative frequency graphs have been moved after box plots as we feel that it is better to introduce quartiles first to provide:

- a gentler introduction to percentiles
- more motivation for *using* cumulative frequency graphs.

In the "Standard deviation" section, we have moved the manual calculations of the standard deviation from the book to an online Activity (page 391). Students will almost always calculate this statistic using technology, so we see little value in included a worked example *in addition to* the manual calculation in the theory on the previous page.

A significant proportion of the material for "The normal distribution" section has been adapted from the corresponding material in our Diploma Programme books.

CHAPTER 19: BIVARIATE STATISTICS

A Scatter graphs
B Correlation
C Pearson's correlation coefficient r
D Line of best fit by eye
E Linear regression

Keywords:

- bivariate data
- bivariate statistics
- correlation
- dependent variable
- extrapolation
- independent variable
- interpolation
- least squares regression line
- linear model
- linear regression
- line of best fit by eye
- mean point
- Pearson's correlation coefficient
- pole
- scatter graph

The "Scatter graphs" section has been adapted to place a greater emphasis on the construction of scatter graphs and interpretation of the data as a whole. We felt that identifying individual points did not fit the overall theme of the chapter, and students would already be familiar with identifying points on a number plane from their studies of coordinate geometry.

Material on the coefficient of determination r^2 has been removed as it is not part of the course. Exercise questions have been adjusted to use the r-value for measuring the correlation between the variables instead.

Most of the skills and concepts required for the "Capture-recapture method" Global Context on page 416 are covered in Chapters 17 and 18 ("Probability" and "Statistics" respectively). However we have placed it in this chapter as one of the later questions requires linear regression.

CHAPTER 20: RELATIONS AND FUNCTIONS

A Relations and functions
B Function notation
C Domain and range
D Sign diagrams
E Transformations of graphs
F The absolute value function
G Composite functions
H Inverse functions

Keywords:

- absolute value
- composite function
- critical value
- domain
- function
- function notation
- function value
- image
- interval notation
- inverse function
- modulus
- natural domain
- range
- reflection
- relation
- scale factor
- sign diagram
- stretch
- transformation
- translation

The purpose of this chapter is to put in place the terminology and structures that will be encountered when studying quadratic functions, exponential functions, and trigonometric functions in later chapters.

In this edition, we have presented domain and range in their own section to give them more emphasis, rather than being part of the Relations section. Section A now focuses on distinguishing between a relation and a function. This structure mirrors what is done in the corresponding chapters of the Diploma Programme courses. This also allows us to place the subsection about the natural domain of a function in Section C (Domain and range) rather than Section B (Function notation).

Section D (Sign diagrams) has been moved here from Chapter 22 (Inequalities) in the previous edition. We have included an Investigation in which students discover how the form of the factors of a function affect its sign diagram. Sign diagrams will be used to solve quadratic inequalities in Chapter 22 (Quadratic functions), and to describe stationary points in Chapter 25 (Differential calculus).

Section E (Transformations of graphs) has been added in this edition. Students apply translations, stretches, and reflections to a function to obtain the graph of a related function. This section has been added since the Transformation geometry chapter has been removed in this edition.

In the Discussion at the end of Section E, students should conclude that:

- No points are invariant when a function is translated, since all points move the same distance in the same direction.
- When a function is stretched vertically or reflected in the x-axis, points on the x-axis are invariant.
- When a function is stretched horizontally or reflected in the y-axis, points on the y-axis are invariant.

Sections G (Composite functions) and H (Inverse functions) are not required by the MYP Framework, and have been placed online. However, students intending to study either of the HL courses in the Diploma Programme would benefit from completing these sections.

CHAPTER 21: QUADRATIC FUNCTIONS

A Quadratic functions
B Graphs of quadratic functions
C Using transformations to graph quadratics
D Axes intercepts
E Axis of symmetry
F Vertex
G Finding a quadratic function
H Problem solving
I Quadratic inequalities

Keywords:

- axis of symmetry
- completed square form
- completing the square
- maximum value
- minimum value
- parabola
- quadratic function
- quadratic inequality
- turning point
- vertex
- x-intercept
- y-intercept

In Section B, we have added an introductory Investigation in which students use the geometric definition to generate a parabola. There is also an Investigation at the end of the section which explores one of the properties of parabolas. Students should find that all of the rays reflected off the parabola pass through a single point, which is the focus of the parabola.

In Section C (Using transformations to graph quadratics), rather than having a large Investigation to explore the graphs of quadratic functions, we can use the work done on transformations in the previous chapter to first establish some basic ideas about the shape and position of $y = (x - h)^2 + k$ and $y = -x^2$. We then investigate the effect of the value of a on the shape of the graph, and the direction in which it opens.

In Questions **7** and **8** of Exercise 21C.1, students must verify the completed square form of each quadratic function, then use this form to sketch the graph. In Section C.2, students must convert the quadratic functions into completed square form themselves.

The material which was in Section 20C.2 (The discriminant and the quadratic graph) in the previous edition has been turned into an Activity which appears at the end of Section D.

Section G (Finding a quadratic function) has been added in this edition. Finding a function based on information about its graph is an important modelling skill which students may find useful in later years, especially those looking to study one of the Mathematics: Applications and Interpretation courses in the Diploma Programme.

The "Quadratic optimisation" section has been renamed "Problem solving" (Section H), since students are asked to solve problems other than identifying the optimum point. The Global Context at the end of this section provides some interesting examples of the use of parabolas and circles in the construction of arches.

Section I (Quadratic inequalities) has been moved here from Chapter 22 (Inequalities) in the previous edition.

CHAPTER 22: NUMBER SEQUENCES

A Number sequences
B Arithmetic sequences
C Geometric sequences
D Sequences in finance
E Series
F Arithmetic series
G Geometric series

Keywords:

- arithmetic sequence
- arithmetic series
- common difference
- common ratio
- compound interest
- converge
- depreciation
- diverge
- explicit formula
- finite geometric series
- general term
- geometric sequence
- geometric series
- infinite geometric series
- initial condition
- nth term
- number sequence
- recurrence relation
- sequence
- series
- term

Section A provides an introduction to the notation and terminology associated with number sequences. We describe sequences by listing the terms, writing the sequence in words, or by writing an explicit formula or recurrence relation.

Although recurrence relations are not mentioned explicitly in the MYP Framework, we inherently use them when describing a sequence in words, so there is value in learning to describe them algebraically. In the Discussion in Section A, students should conclude that we can also write a description for a sequence in words which is linked to the explicit formula. For example, a sequence with explicit formula $u_n = 3n$ can be described in words as "the nth term of the sequence is equal to 3 times n".

Sections B (Arithmetic sequences) and C (Geometric sequences) will provide a useful introduction to the sequences work done in any of the Diploma Programme courses.

We have converted the "Sequences in finance" Investigation from the previous edition into a section of its own (Section D). This will replace the sections on compound interest and depreciation that were in Chapter 18 (Exponential functions and logarithms) in the previous edition. Studying them here, in the context of sequences, will better mirror what is done in the Diploma Programme courses.

The main difficulty here is understanding that we consider the initial condition to represent the "zeroth" term u_0. We do this so that the situation after n time periods is represented by u_n, rather than u_{n+1}.

The work done here will also help in understanding exponential functions in the following chapter. Even though exponential functions deal with continuous time intervals, the idea of exponential growth is more intuitively described by thinking in terms of discrete time intervals, such as what is done here for compound interest and depreciation. For example, an exponential function $f(x) = 5 \times 1.2^x$ tells us that as x increases by 1, the value of the function increases by 20%.

The Sections E to G are marked in dark blue, as they are not explicitly part of the MYP Framework. However, they are also an important part of the Diploma Programme courses, so there would be value in students getting an introduction to arithmetic and geometric series here. Students will also encounter the series notation in their study of calculus in later years.

This chapter contains two Global Context projects. Although both projects involve number sequences, they are quite different to each other. The golden ratio Global Context at the end of Section A guides students through some useful properties of the golden ratio, as well as discussing its occurrence in the world around us. This project discusses the link between the Fibonacci sequence and the golden ratio, so students may benefit from completing the preceding Investigation about the Fibonacci sequence. The spider webs Global Context at the end of Section G examines the geometric properties of spider webs constructed according to particular rules. In order to determine an "optimal" design for a spider web, students must analyse a large table of data, which is likely to produce some surprising results.

CHAPTER 23: EXPONENTIALS AND LOGARITHMS

A Exponential functions
B Graphs of exponential functions
C Exponential equations
D Exponential growth
E Exponential decay
F Common logarithms
G Laws of logarithms
H Using logarithms
I Logarithms in other bases

Keywords:

- common logarithm
- exponential decay
- exponential equation
- exponential function
- exponential growth
- horizontal asymptote
- logarithm
- logarithm laws
- y-intercept

This will be the first time students have encountered exponential functions. Students should understand that, while linear functions are characterised by a quantity changing by a constant *amount* each time period, exponential functions are characterised by a quantity changing by a constant *percentage* each time period. It may help students to look back at the work on sequences in finance, and see that the formula for the value of a compound interest account, or the value of a depreciating item, was an exponential function, which increased or decreased by a certain percentage each time period. What we are doing here is extending the function to one that applies to *all* values within the domain, rather than at discrete time intervals.

Now that a section about transformation of graphs has been added, we no longer need a series of investigations about using transformations to graph exponential functions. We simply apply the general principles we studied to exponential functions in Section B.

In this edition, we have moved exponential equations ahead of exponential growth and decay. The exponential equations section now includes practice at solving exponential equations using technology. This can either be done graphically, or by using the Solver function of the calculator. This can then be used in exponential growth and decay to answer questions such as "How long will it take for the population to reach 500?".

The material about compound interest and depreciation that was in Sections 18D and 18E in the previous edition have been moved to Chapter 22, and are studied in the context of sequences. This not only follows the structure of the Diploma Programme courses, but also gives a more logical progression. We first consider contexts involving discrete time units, before moving to growth and decay contexts such as population or temperature, in which the function can be evaluated at *any* time within a continuous range.

The work on logarithms has been expanded from what was in the previous edition, and is presented in four separate sections, rather than as subsections within a single section.

In Section F, students are introduced to the concept of a logarithm, and hopefully get a feel for how logarithms work.

In the Discussion at the end of Section F, students should find that $\log(10^n)$ is equal to n for any real n, and that $10^{\log n}$ is equal to n for any $n > 0$. Once the concept of a logarithm is understood, these results follow from the definition of a logarithm. Students should also find that $\log 0$ does not exist, since we cannot raise 10 to any power to obtain the number 0.

In Section I (Logarithms in other bases), we have restricted the bases used to integers, avoiding impractical bases such as $\frac{1}{8}$ or $\sqrt{2}$.

CHAPTER 24: ADVANCED TRIGONOMETRY

Keywords:

- amplitude
- complementary angle identities
- cosine
- double angle identities
- maximum point
- mean line
- minimum point
- negative angle identities
- period
- periodic function
- principal axis
- Pythagorean identity
- sine
- tangent
- trigonometric function
- trigonometric identity
- unit circle

In this edition, we have removed the material on radian measure, as it is not part of the MYP Framework. All of the work done in this chapter is therefore presented in terms of degrees.

We start this chapter by extending the definition of trigonometric ratios for *all* angles. If students have understood how we defined trigonometric ratios for obtuse angles in Chapter 16, then extending the definition further should not be problematic.

We feel that the trigonometric ratios for multiples of $30°$ and $45°$ follow more logically from the definition of the trigonometric ratios in Section A, so in this edition we have moved this work up to Section B.

In Section D, we have added an Investigation in which students use the trigonometric ratios found in Section B to generate the graphs of $y = \sin x$ and $y = \cos x$ for themselves. This should give students a greater understanding of the graphs' features, including their maximum and minimum values, and their periodic nature.

In Section E, we use the work done in Chapter 20 to generate transformations of $y = \sin x$ and $y = \cos x$. Given that we have special terminology to describe periodic functions such as amplitude, period, and principal axis, the transformed functions are generally described using this terminology, rather than explicitly referencing the transformations involved.

In Section G, students solve trigonometric equations. Most equations require knowledge of the trigonometric ratios for multiples of $30°$ and $45°$. It is important that students understand how to generate all of the solutions in the given domain. For example, they may need to add or subtract multiples of $360°$ to the solutions which can be read from the unit circle.

Sections H (Negative and complementary angle identies) and I (Double angle identities) are marked in dark blue as they are not required by the MYP Framework. However, students should be encouraged to complete these sections if time permits, especially students looking to study either of the Mathematics: Analysis and Approaches courses in the Diploma Programme.

CHAPTER 25: DIFFERENTIAL CALCULUS

A Limits
B Finding the gradient of a tangent
C The derivative function
D Differentiation from first principles
E Rules for differentiation
F Finding the equation of a tangent
G Stationary points

Keywords:

- calculus
- converge
- derivative
- derivative function
- differential calculus
- differentiation
- differentiation from first principles
- gradient function
- instantaneous rate of change
- limit
- local maximum
- local minimum
- stationary inflection
- stationary point
- tangent
- turning point

In this edition, the calculus material has been split into two chapters: Differential calculus, and Integration. This material is not part of the MYP Framework, so we have marked it dark blue, and placed it online. However, calculus plays a large role in all of the Diploma Programme courses, so if students are able to complete these chapters in MYP 5, they would be at a huge advantage going into their Diploma Programme courses.

This chapter essentially covers what was in Sections A to E of Chapter 25 (Introduction to calculus) in the previous edition. This work has been restructured to match the structure used in the calculus chapters in the Diploma Programme courses.

The work that was previously in Section A (Tangents) about estimating the gradient of a tangent from a graph, and using quadratic theory to find the gradient of a tangent, has been converted into two Activities before Section A, since we felt that it was not really the focus of the chapter. These Activities motivate the idea that, in order to find the *exact* gradient of a tangent for *any* function, we need the mathematical principle called a limit, which is explored in Section A.

In Section B (Finding the gradient of a tangent), we use limits to find the gradient of a tangent to a function at a particular point. This work appeared in Section 25B.2 of the previous edition.

The material in Section C (The derivative function) is mainly new, and is aimed at familiarising students with the notation surrounding the derivative function, such as $f'(x)$ or $f'(2)$. The material in the section of the same name in the previous edition was concerned with finding the derivative function from first principles. This work is done in Section D.

We have added Section F (Finding the equation of a tangent) in this edition. We feel that once students can find the gradient of a tangent, it is logical to take the next small step to find its equation.

CHAPTER 26: INTEGRATION

A The area under a curve
B Integration
C Rules for integration
D The definite integral
E The Riemann integral

Keywords:

- antiderivative
- antidifferentiation
- constant of integration
- definite integral
- integral
- integral calculus
- integration
- Riemann integral

This chapter essentially covers what was in Sections F to H of Chapter 25 (Introduction to calculus) in the previous edition.

The Opening Problem should lead students to the idea that we can estimate the area under a curve using rectangles to approximate the region. In Section A, we extend this idea and see how limits can be used to find the area exactly.

In Section B, we define integration as the reverse process of differentiation, and use this to find the integral of particular functions. This work was covered in Section 25G.1 in the previous edition. This process leads to the establishment of some general rules for integration (Section C), which was covered in Section 25G.2 in the previous edition. In this edition we have distinguished between the rules which follow directly from the rules of differentiation, and the more general rules about integrating the sum or difference of two functions, or a function multiplied by a scalar.

We have divided the material that was in Section 25H (The definite integral) in the previous edition into two sections. Section D defines the definite integral and introduces the notation associated with it. Section E looks at the interpretation of the definite integral as the area under the curve.

CHAPTER 27: VECTORS

A Vectors and scalars
B Geometric vector addition
C Geometric vector subtraction
D Geometric scalar multiplication
E Vectors in component form
F Position vectors
G The magnitude of a vector
H Operations with vectors
I Parallelism
J The scalar product of two vectors
K The angle between two vectors

Keywords:

- component form
- displacement vector
- dot product
- equal vectors
- negative vector
- parallel vectors
- position vector
- scalar
- scalar multiplication
- scalar product
- vector
- zero vector

Vectors are not part of the MYP Framework, so we have marked this chapter as dark blue, and placed it online in this edition. This chapter will be useful preparation for students intending to study either of the HL Mathematics courses in the Diploma Programme.

In the Discussion at the end of Section A, students should conclude that a vector of length 0 has no direction, and such a vector could be represented as a dot. The zero vector is explored further in Section B.

In this edition, we have added a section about geometric scalar multiplication of vectors (Section D), to place this operation in line with the other operations.

Students should get a good understanding of the vectors in a geometric sense in Sections A to D, before the component form is introduced in Section E. It would be helpful to compare this work to what is done in coordinate geometry. In coordinate geometry, we develop formulae for the distance between points, and the midpoints and gradients of line segments, based on the coordinates of the points. This allows us to perform calculations without having to draw a Cartesian plane each time. In the same way, the component form of vectors allow us to perform operations with vectors without having to represent them geometrically.

In this edition, we have presented a short section which introduces the scalar product of two vectors (Section J), followed by a section which explains how the scalar product can be used to find the angle between two vectors (Section K).

We have removed three-dimensional vectors in this edition, as we feel there is already sufficient material here to provide a useful introduction to vectors.

CHAPTER 28: COUNTING AND PROBABILITY

- **A** The product principle
- **B** The sum principle
- **C** Factorial notation
- **D** Permutations
- **E** Combinations
- **F** Probabilities using permutations and combinations

Keywords:

- combination
- counting problem
- factorial number
- permutation
- product principle
- sum principle

This chapter is not part of the MYP Framework, and has been marked in green. This indicates that this chapter is useful preparation for students intending to study the Mathematics: Analysis and Approaches HL course in the Diploma Programme.

Sections A and B introduce students to the product and sum principles. Students should be able to identify when the numbers of possibilities should be multiplied, and when they should be added.

In Section D, students are introduced to permutations. These are selections in which the order of selection *is* important. Although the formula for the number of permutations is given in the factorial form $\frac{n!}{(n-r)!}$, students may find that the form $n \times (n-1) \times (n-2) \times .... \times (n-r+1)$ is more useful, and more intuitive as it flows directly from the product principle. The main purpose of the factorial formula comes in Section E, where the formula for the number of combinations is found by dividing the number of permutations by $r!$. This occurs because, for each combination, there are $r!$ corresponding permutations formed by ordering the r selected objects.

In Section F, we find probabilities using permutations and combinations. In this section, we calculate the number of relevant permutations or combinations as in Sections D and E, but then must divide by the total number of possible outcomes to find the relevant probability. Since permutations and combinations are combined here, students must be able to decide which must be used in each question. Some of the questions can be done using either combinations or permutations, however in these cases students must be consistent in their approach when calculating number of outcomes in the event, and the total number of possible outcomes.