Mathematics for Australia Year 10 2nd edition

Chapter summaries

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ASSUMED KNOWLEDGE 1: NUMBER

A Exponent notation
B Order of operations
C Absolute value
D Percentage
E Percentage change

Keywords:

- absolute value
- base
- BEDMAS
- exponent
- exponent notation
- index
- multiplier
- percentage
- power

This online chapter contains some material on order of operations which was in Chapter 1 of the previous edition, as well as some revision on exponent notation, absolute value, and percentage.

Students who need this help should be encouraged to study the chapter out of classroom time, since most students will not need this chapter, and classroom time spent on this chapter is likely to come at the expense of time spent on the more advanced chapters at the end of the year.

ASSUMED KNOWLEDGE 2: ALGEBRA

A Algebraic notation
B Writing expressions
C Algebraic substitution
D The language of algebra
E Collecting like terms
F Algebraic products

Keywords:

- coefficient
- constant
- equation
- evaluate
- exponent notation
- expression
- factor
- like terms
- product
- product notation
- substitute
- term
- variable

This online chapter has been included for students who feel they need extra help with the basics of algebra. It is essentially revision of the work done in Chapter 2 (Algebra: Expressions) of the Year 9 book. As with Assumed Knowledge 1, this chapter should be done out of classroom time if possible.

CHAPTER 1: EXPONENTS

A Exponent laws
B Scientific notation

Keywords:

- base
- exponent
- exponent laws
- exponent notation
- index
- power
- scientific notation
- standard form

The material in this chapter is adapted from Sections C and D from the "Operations with numbers" chapter in the previous edition. From that chapter, Section A (Operations with integers) has been removed, and Section B (Order of operations) has been moved to Assumed Knowledge 1.

In this edition, "indices" have been renamed "exponents" due to the change in terminology in the most recent syllabus update.

It is assumed that students will be familiar with exponent notation at this point, and Section A moves straight into the exponent laws. Students who need more practice with exponent notation can access some revision work in Assumed Knowledge 1 Section A.

In Section B, we have adjusted our approach to explaining scientific notation, to give students more guidance in how to choose their values of a and k in $a \times 10^k$.

CHAPTER 2: ALGEBRA: EXPANSION

A The distributive law
B The product $(a+b)(c+d)$
C The difference between two squares
D The perfect squares expansion
E Further expansion
F The binomial expansion

Keywords:

- binomial
- binomial expansion
- difference between two squares
- distributive law
- expansion
- factorisation
- FOIL rule
- perfect squares expansion

In this edition, we have split algebraic expansion and factorisation into two separate chapters. This gives us the opportunity to provide a more complete coverage of algebraic expansion than what was given in the previous edition. Given that students will have only seen most of this material for the first time in Year 9, it seems reasonable to spend a little more time in Year 10 to reinforce this work.

In Section C, some of the expansions involve working with surds. Although this comes before Chapter 8 (Surds and other radicals), it only involves very intuitive calculations such as $(\sqrt{2})^2 = 2$ based on the definition of a surd, which students should be familiar with from previous years.

In Section E (Further expansion), it may help students to recognise that they have already encountered the idea of "multiplying each term in the first bracket by each term in the second bracket", since that is what happens when applying the FOIL rule to $(a+b)(c+d)$ in Section B.

Section F (The binomial expansion) will now be new to students, since the binomial expansion was removed from Year 9 in this edition. The Investigation on Pascal's triangle at the end of the section gives students a hint towards generating a general binomial expansion, which students may explore further in Years 11 and 12.

CHAPTER 3: ALGEBRA: FACTORISATION

A Common factors
B Difference between two squares factorisation
C Perfect squares factorisation
D Expressions with four terms

E Factorising $x^2 + bx + c$
F Miscellaneous factorisation

Keywords:

- common factors
- difference between two squares
- factorisation
- fully factorised
- linear factors
- perfect squares
- quadratic trinomial
- sum and product method

In this edition, algebraic factorisation has been separated from expansion into its own chapter. As with expansion, this will allow us to provide a more complete coverage of the factorisation material.

To emphasise that factorisation is the reverse process of expansion, students should be reminded that they can check their factorisations by expanding their answer.

Students should be familiar with the material in Sections A to C from Year 9, so these sections can be worked through quickly if needed.

In the Discussion in Section E, students should find that:

- If the sum and product of two numbers are both positive, the numbers must both be positive.
- If the sum and product of two numbers are both negative, the numbers are opposite in sign, and the negative number has the largest absolute value.
- If the sum is positive and the product is negative, the numbers are opposite in sign, and the positive number has the largest absolute value.
- If the sum is negative and the product is positive, the numbers must both be negative.

In Section F, students will need to choose which factorisation method to use. Students may wish to produce their own summaries describing when each method is suitable.

CHAPTER 4: SETS

A Sets
B Complement of a set
C Intersection and union
D Special number sets
E Interval notation

Keywords:

- complement
- complementary sets
- disjoint
- element
- empty set
- equal sets
- finite set
- infinite set
- integers
- intersection
- interval notation
- irrational numbers
- member
- natural numbers
- negative integers
- positive integers
- rational numbers
- real numbers
- set
- subset
- union
- universal set

This chapter has been added in this edition. The chapter contains some useful work regarding special types of numbers and interval notation, which are used frequently throughout the book, so it is helpful to address this work explicitly.

This material is often presented alongside Venn diagrams as a single chapter, however, as was the case in Year 9, we felt the best approach was to split Sets and Venn diagrams into two chapters, with "Linear equations and inequalities" between them. This way, interval notation can be introduced in Sets before it is used in linear inequalities, and linear equations can be used to solve problems involving Venn diagrams.

When discussing the union "A or B", it is important to emphasise that elements in both A and B are included in the union. This is a good opportunity to discuss how words can be used differently in mathematics than they are in everyday use, as "or" is often used to mean "one or the other, but not both" in everyday use.

In this edition, we have changed the definition of natural numbers to include 0. Although either interpretation is accepted in the Australian Curriculum glossary, this appears to be the most commonly used definition in other curriculums around the world.

We have endeavoured to extend what was done in the Year 9 sets chapter by giving more opportunities to explore the concepts of finite and infinite sets. In the Discussion at the end of Section D, students should consider that, when we have two *finite* sets A and B where A is a subset of B, it is clear that there are more elements in B than in A. However, it is less clear when A and B are both infinite sets! It is tempting to say that there are more elements in $\mathbb{Z}$ than in $\mathbb{Z}^+$, since $\mathbb{Z}$ contains all the elements of $\mathbb{Z}^+$, as well as some extra elements. But does it make sense to say that one infinite set has *more* elements than another infinite set? A potentially more illuminating example may be if A is the set of even integers, and B is the set of positive integers. Clearly A is a subset of B, but one could easily generate each element of A by multiplying each element of B by 2! Does this mean that, in some sense, they have the same number of elements?

A similar question occurs in the second dot point. Since the interval of numbers from 0 to 1 appears smaller than the interval of numbers greater than 1, it seems logical that there are more numbers greater than 1 than there are numbers between 0 and 1. However, for each number greater than 1, there is a corresponding number between 0 and 1, which is found by taking the reciprocal of the original number. Does this correspondence mean that there are the same number of real numbers between 0 and 1 as there are greater than 1?

When dealing with these questions, students should be reminded that rules that apply to finite sets do not necessarily extend to infinite sets.

CHAPTER 5: ALGEBRAIC FRACTIONS

- **A** Evaluating algebraic fractions
- **B** Simplifying algebraic fractions
- **C** Multiplying algebraic fractions
- **D** Dividing algebraic fractions
- **E** Adding and subtracting algebraic fractions

Keywords:

- algebraic fraction
- evaluate
- lowest common denominator
- lowest terms
- rational expression
- reciprocal

The structure of this chapter remains largely unchanged from what occurred in the previous edition. The main difference is that multiplication and division of algebraic fractions have been split into separate sections.

Placing this chapter after we have studied factorisation allows us to factorise the numerator and denominator of an algebraic fraction. This helps in cancelling common factors when simplifying, multiplying, or dividing algebraic fractions.

In Section E, we have avoided using the term "simplify" as much as possible, as this may be ambiguous. For example, in some questions students must write the sum of two fractions as a single fraction, whereas in other questions students must take a single fraction such as $\frac{x+9}{3}$, and write it as the sum of two parts. It is therefore unclear which form is the "simplest" in this case. Instead we have been more explicit about what the student should do in each question.

This may be a good opportunity to discuss the merits of the term "simplify", and to help students understand that when we manipulate an algebraic expression, we are turning it into a different form. Whether this new form is "better" or "simpler" may depend on what we are trying to do with the expression.

CHAPTER 6: LINEAR EQUATIONS AND INEQUALITIES

- **A** Linear equations
- **B** Equations with fractions
- **C** Problem solving
- **D** Linear inequalities
- **E** Problem solving with inequalities

Keywords:

- algebraic equation
- double inequality
- equal sign
- equation
- inequality
- inverse operations
- left hand side
- linear equation
- linear inequality
- lowest common denominator
- right hand side

Section A is largely revision of the work done on solving linear equations in previous years, and may be skipped through more quickly if students are comfortable with the material.

In Section B, students use the work done in the previous chapter to solve equations involving algebraic fractions.

In Section C (Problem solving), we have added some questions where it is useful to present the information in a table, in order to translate the problem into an equation.

Since linear inequalities are now introduced in Year 9, there is less background information about linear inequalities in Section D than there was in the previous edition, and we move more quickly towards solving linear inequalities. When solving linear inequalities, students can use the work done in Chapter 4 to write the solutions in interval notation, and to graph the solution on a number line.

In this edition, we include double inequalities, such as $5 < 3x - 4 < 17$. When solving double inequalities, students must remember to write the expressions in the solution from smallest to largest. So, if students have obtained the solution $8 > x > 5$, they must write this as $5 < x < 8$. Students should not think of this step as "reversing the inequality signs" in the same way as when they multiply or divide by a negative number. Instead, it is more about rewriting the solution in the correct form.

In the Discussion at the end of Section D, students should find the double inequalities such as $x + 1 \leqslant 3x - 5 < 10$ can be solved by treating them as two separate inequalities, and then combining the solutions. For example, the solution to $x + 1 \leqslant 3x - 5$ is $x \geqslant 3$, and the solution to $3x - 5 < 10$ is $x < 5$. So, the solution to the double inequality occurs where *both* of these are true, which is the interval $3 \leqslant x < 5$.

CHAPTER 7: VENN DIAGRAMS

A Venn diagrams
B Venn diagram regions
C Numbers in regions
D Problem solving with Venn diagrams

Keywords:

- complement
- disjoint
- intersection
- set identity
- subset
- union
- Venn diagram

This chapter has been added in this edition. We felt that, in order to use Venn diagrams in the Probability chapter, it would be preferable to introduce Venn diagrams in their own right earlier in the book. As was done in Year 9, Venn diagrams has been separated from the Sets chapter, and placed after linear equations, so that students can use their linear equation solving skills to find unknown numbers in regions on Venn diagrams.

In Year 10, the work is extended to consider more Venn diagrams with three sets, and to prove some set identities.

In Section D, the problems involving 3 sets may appear challenging, but the combination of information given is such that the number of elements in all three sets can be found immediately, and the rest of the regions can be deduced quite easily from the remaining information.

CHAPTER 8: SURDS AND OTHER RADICALS

A Radicals
B Properties of radicals
C Simplest surd form
D Power equations
E Operations with radicals

Keywords:

- cube root
- nth root
- power equation
- radical
- simplest surd form
- square root
- surd

In Section A, students perform calculations based purely on the definition of the square root, such as $\sqrt{5} \times \sqrt{5} = 5$. In Section B, students use properties of square roots, such as $\sqrt{a} \times \sqrt{b} = \sqrt{ab}$, to perform calculations.

In Section C, more able students may be asked to consider situations where simplest surd form is useful, and situations where it is better to leave the surd in its original form.

In this edition, we have added Section D (Power equations). We feel that it is important to consider power equations, as they are useful for solving problems in topics such as measurement, proportion, and similarity, but are rarely addressed in their own right.

In the Discussion at the end of Section D, students should find that, when both sides of an equation are squared, we often introduce an extra solution. There are often good reasons to square both sides of an equation, but we need to be aware that this may create extra solutions, and check whether the solutions we obtain all satisfy the original equation.

The sections "Adding and subtracting radicals" and "Multiplications involving radicals" in the previous edition have been combined into the single section "Operations with radicals". Since the rules for operating with radicals are identical to those for algebra, we felt there was little to be gained by keeping these sections separate.

CHAPTER 9: PYTHAGORAS' THEOREM

A Pythagoras' theorem
B Pythagorean triples
C Problem solving
D Circle problems
E The converse of Pythagoras' theorem

Keywords:

- chord
- converse of Pythagoras' theorem
- Pythagoras' theorem
- Pythagorean triple
- radius
- semi-circle
- tangent

In this edition, we have changed the order of the sections, so that the converse of Pythagoras' theorem is done at the end. This allows us to consider the non-contextual and contextual applications of Pythagoras' theorem first, before considering the converse.

In this edition, we have added some three-dimensional problems to Section C (Problem solving). These problems usually involve using Pythagoras' theorem twice, but we do not feel there is a great conceptual leap in solving problems in three dimensions.

Classes which are using the 10A book in parallel may notice that this chapter contains a section on circle problems (Section D) which is not in the 10A book. This is because these types of questions appear in a later chapter (Chapter 18: Circle geometry) which is not in the Year 10 book.

CHAPTER 10: FORMULAE

A Formula construction
B Substituting into formulae
C Rearranging formulae
D Rearrangement and substitution
E Predicting formulae

Keywords:

- formula
- inverse operations
- rearrange
- subject
- substitution

In this edition, we have moved the Formulae chapter before the Measurement chapter, as we felt that students will benefit from doing this work before encountering the measurement formulae. As a result, we have removed any questions referring to measurement formulae, as these will be addressed in the Measurement chapter. In their place, we have included more questions based on physics formulae, as specified in the new Australian Curriculum.

We have also included more questions in which students must solve a power equation to find an unknown variable. The work done in Chapter 8 should help with this.

In Section D, students are asked to rearrange the formula to make a particular variable the subject, and then use substitution to evaluate that variable in different circumstances. The students are always asked to evaluate the variable multiple times, as this serves to highlight the advantage of rearranging the formula first. If the formula is rearranged first, the rearrangement only needs to be done once, rather than for each time the variable must be evaluated.

In Section E, students observe the first few numbers in a sequence, identify the pattern in the sequence, and use this information to generate a formula for the nth term.

CHAPTER 11: MEASUREMENT

A Length and perimeter
B Area
C Surface area
D Volume
E Capacity

Keywords:

- arc length
- area
- capacity
- centimetre
- circumference
- hectare
- kilometre
- length
- metre
- millimetre
- perimeter
- surface area
- tapered solid
- volume

Now that this chapter occurs after the Formulae chapter, we have included more questions in which students must find an unknown which is not the subject of the formula, for example, finding the radius of a circle given its circumference.

In this edition, all of the measurement formulae given here, including the surface area and volumes of tapered solids and spheres, were introduced in Year 9. This means that in Year 10 there is less need to derive each of these formulae, as this was largely done in Year 9, where appropriate.

This approach should allow classes to move through measurement much faster than they would have in previous years. We feel this is important, as there is a lot of other more rigorous content which must be covered at Year 10.

CHAPTER 12: FINANCIAL MATHEMATICS

A Business calculations
B Appreciation and depreciation
C Simple interest
D Compound interest

Keywords:

- appreciation
- break even
- compound interest
- compound interest formula
- cost price
- depreciation
- discount
- future value
- goods and services tax
- inflation
- interest
- interest rate
- loss
- marked price
- mark-up
- multiplier
- per annum
- present value
- principal
- profit
- selling price

- simple interest
- simple interest formula

In this chapter, students apply their knowledge of percentages in financial contexts such as discount and mark-up, appreciation and depreciation, and simple and compound interest.

Section A has undergone a significant restructure from the previous edition. Whereas Section A was split into 5 subsections in the previous edition, here we have presented mark-up, discount, profit, loss, and tax in a single section, noting that each is an application of percentage change.

In Activity 1, we ask students to take the very general percentage change formula "old amount $\times$ multiplier $=$ new amount", and specify what "old amount" and "new amount" refer to in each context. For example, when considering mark-up, we have "selling price $=$ marked price $\times$ multiplier", and when considering loss, we have "selling price $=$ cost price $\times$ multiplier". We hope that this approach helps students see that each context involves the same processes, and that students should not feel that they need to memorise a distinct set of formulae for each context.

In Section C, given that the students have only recently completed the Formulae chapter, we do not feel it is necessary to present a worked example on each of the rearrangements of the simple interest formula, especially since each rearrangement is essentially equivalent. Instead, students are asked in Question **8** to rearrange the simple interest formula to make each other variable the subject, and then the remaining questions in the exercise require one of those arrangements.

CHAPTER 13: QUADRATIC EQUATIONS

A Equations of the form $x^2 = k$
B The null factor law
C Solving by factorisation
D Completing the square
E The quadratic formula
F Problem solving

Keywords:

- completed square form
- completing the square
- discriminant
- null factor law
- quadratic equation
- quadratic formula

In Section A (Equations of the form $x^2 = k$), students may notice that simple versions of these equations were solved in Section 8D (Power equations). Here we use the same principle to solve more complicated equations such as $(3x-2)^2 = 10$.

In Section C.2, students should recognise that equations such as $x^2 - 9 = 0$, which can be solved by difference between two squares factorisation, could also be solved by rearranging it to $x^2 = 9$. However, for more complicated equations, using difference between two squares factorisation is more efficient.

In the Discussion at the end of Section D, students should recognise that the method of "completing the square" can be used when the coefficient of x^2 is not 1. To do this, we divide the whole equation through by that coefficient of x^2, so that the coefficient of x^2 becomes 1. This approach is used when completing the square on the general quadratic equation on the next page.

Students should find that solving quadratic equations by completing the square is quite tedious, and that by applying completing the square to the general quadratic equation $ax^2 + bx + c = 0$, we can obtain the quadratic formula for the solution to the equation in terms of a, b, and c, without having to perform all of the steps each time.

In the Discussion in Section E, students should be able to solve the quadratic equation using factorisation, completing the square, and using the quadratic formula. They should also find that using factorisation is the quickest method in this case. This is a good opportunity to explain that the quadratic formula *can* be used for any quadratic equation, however it is worth checking whether the quadratic equation can be solve by factorisation first, since this method will be much quicker.

In this edition, similarity has been moved later in the book, and now appears after quadratic equations. For this reason, the questions involving similar triangles have been removed from Section F, and now appear in the Congruence and similarity chapter.

CHAPTER 14: COORDINATE GEOMETRY

A The distance between two points
B Midpoints
C Gradient

D Parallel and perpendicular lines
E Using coordinate geometry
F 3-dimensional coordinate geometry

Keywords:

- Cartesian plane
- coordinates
- distance formula
- gradient
- gradient formula
- midpoint
- negative reciprocals
- number plane
- ordered pair
- origin
- parallel lines
- perpendicular lines
- quadrant
- x-axis
- X-axis
- x-coordinate
- y-axis
- Y-axis
- y-coordinate
- Z-axis

In this edition, as with in the Year 9 book, the material involving the equations of lines has been moved to its own chapter "Straight lines" (Chapter 15). In the content that remains, students explore the Cartesian plane, including distance between points, midpoints, and gradients. The work in this chapter provides students the tools to describe straight lines in the Cartesian plane in the following chapter.

In the Discussion in Section D, students should find that the rule for gradients of perpendicular lines does not apply to horizontal and vertical lines, since vertical lines have undefined gradient. The idea that vertical and horizontal lines are perpendicular should be intuitive to students, and gradient is not a very useful tool in this case.

Knowledge of the gradient of parallel and perpendicular lines allows us to verify and prove geometric facts in Section E (Using coordinate geometry), which is a new section for this edition.

Section F (3-dimensional coordinate geometry) has also been added in this edition, as it has been included in the Australian Curriculum. Students should be reminded that, although it is harder to visualise coordinates in three dimensions, most of the work that is done (such as finding distances and midpoints) extends fairly logically from what they have seen in two dimensions.

CHAPTER 15: STRAIGHT LINES

A The equation of a line
B Graphing straight lines
C Finding the equation of a line
D Linear inequalities in the Cartesian plane

Keywords:

- axes intercepts
- equation of a line
- general form
- gradient-intercept form
- point-gradient form
- x-intercept
- y-intercept

This new chapter comprises the material about straight lines that was in Sections E to G of the Coordinate geometry chapter in the previous edition. Given that some new material has also been added in this edition, presenting this work as a chapter in its own right allows us to provide a more complete treatment of this material, without making the chapter too large.

In Section A, we have more explicitly introduced the point-gradient form of the equation of a line. From here the equation can be rearranged into gradient-intercept form, or general form. Students find the gradient of a line in general form by rearranging it to gradient-intercept form. However, with sufficient practice, students should be able to quickly find the gradient of a line from its general form.

We have restructured the text in Section C so that, rather than focusing on the form in which to write the equation, we consider the different combinations of information students may be given about a line, and how to find the equation of the line in each case.

Section E (Linear inequalities in the Cartesian plane) has also been added in this edition, since it was added to the Australian Curriculum. To graph a region such as $2x + 5y < 10$, students should first graph the line $2x + 5y = 10$, and then use a test point such as $(0, 0)$ to determine which side of the line the required region lies on.

CHAPTER 16: SIMULTANEOUS EQUATIONS

- **A** Graphical solution
- **B** Solution by substitution
- **C** Solution by elimination
- **D** Problem solving

Keywords:

- elimination
- simultaneous equations
- simultaneous solution
- substitution

When studying simultaneous equations, it is important that students understand the conceptual shift in that our solution takes the form of a value of x *and* y which make both equations true simultaneously.

In Section A, a graphical approach is used. This should allow students to use what they learnt in the previous chapter, and see that by graphing the line corresponding to each equation, the intersection point gives us the solution to the simultaneous equations.

This approach should illustrate to students why some systems have no solutions or infinitely many solutions. However, reading the solution from a graph makes it difficult to find non-integer solutions accurately. This leads to a need for the algebraic approaches outlined in Sections B and C.

CHAPTER 17: CONGRUENCE AND SIMILARITY

- **A** Congruent triangles
- **B** Proof using congruence
- **C** Similarity
- **D** Similar triangles
- **E** Areas and volumes of similar objects

Keywords:

- congruent figures
- congruent triangles
- equiangular
- similar figures
- similar triangles

Since students will have studied congruence in Years 8 and 9, in this edition we have decided to remove the introductory section about congruent figures, and instead move straight into congruence of triangles.

The Investigation in which students discover the information required to uniquely draw a triangle has been moved to Year 9. In Year 10, students are given the criteria under which two triangles can be shown to be congruent.

In the Discussion in Section C, students should find that all squares are similar, and all circles are similar, but not all triangles are similar.

It should be highlighted that, for figures in general to be similar, both the figures must be equiangular *and* the side lengths must be in proportion. In particular, establishing one of these without the other is not sufficient. Questions **4** and **5** highlight that quadrilaterals that are equiangular may not be similar, and quadrilaterals with side lengths in the same ratio may not be similar. However, *triangles* are special in that, if one of these properties is true, the other must also be true. So, to establish two triangles are similar, only one of these properties need be proven. This is what is explored in Section D.

In Section D, some questions which involve solving a quadratic equation have been added. These questions were in the quadratic equations chapter in the previous edition, but needed to be moved here because the quadratic equations chapter is now placed before similarity.

Whereas areas and volumes of similar objects were given separate sections in Year 9, they are considered in one section here. This gives more opportunity to explore the relationships between lengths, surface areas, and volumes of similar three-dimensional objects.

CHAPTER 18: TRIGONOMETRY

- **A** Labelling right angled triangles
- **B** The trigonometric ratios
- **C** Finding side lengths

D Finding angles
E Problem solving
F True bearings

Keywords:

- adjacent side
- angle of depression
- angle of elevation
- cosine
- hypotenuse
- inverse cosine
- inverse sine
- inverse tangent
- opposite side
- sine
- tangent
- trigonometry
- true bearing
- true north

When introducing the trigonometric ratios in Section B, the aim should be not only to familiarise students with the side lengths involved in each trigonometric ratio, but to help them understand that a ratio such as $\sin 57°$ is not just an abstract term, but an actual number whose value can be determined by measuring sides of right angled triangles.

As with the chapter on Pythagoras' theorem, some questions involving three-dimensional objects have been added to Section E.

The work on true bearings (Section F) extends what is done in Year 9 to consider multi-leg journeys, in which students must find the bearing of the end point from the starting point. The bearings of each leg are chosen to create a right angled triangle.

CHAPTER 19: PROBABILITY

A Sample space and events
B Theoretical probability
C Independent events
D Dependent events
E Experimental probability
F Expectation
G Conditional probability
H Simulations

Keywords:

- 2-dimensional grid
- certain event
- combined events
- complementary events
- compound events
- conditional probability
- dependent events
- event
- expectation
- experimental probability
- frequency
- impossible event
- independent events
- probability
- relative frequency
- sample space
- simulation
- theoretical probability
- tree diagram

The contents of this chapter have been restructured to mirror the structure used in Year 9.

We have added Section A (Sample space and events) in this edition. This section considers not only the different ways to represent the sample space of an experiment, but also the definition of an event connected to an experiment, and how the outcomes of a particular event can be highlighted within the sample space. This serves to give this section some more substance, justifying its inclusion at Year 10.

In Section B (Theoretical probability), the method for finding unknown numbers in Venn diagram regions has been updated to reflect the more formal method described in Chapter 7.

In the Puzzle at the end of Section B, students should be able to recognise that an easy way to solve the puzzle is to add 1 to each face of one die (so it is numbered 2 to 7), and subtract 1 from each face of the other die (so it is numbered 0 to 5). However, students who find this solution should be challenged to find a less trivial solution; one that involves the same number appearing on a die more than once.

Section E (Experimental probability) has been added in this edition. Experimental probabilities are estimated based on repeated experiments, as well as from data given in frequency tables and two-way tables.

Section F (Expectation) is likely to be new to students. It is a reasonably intuitive idea, so students should not have trouble with it, but it is an important first step towards some more complex work involving expected value in Years 11 and 12.

Section H (Simulations) has been added to the chapter in this edition. In this section, students use simulations to estimate probabilities in situations where calculating theoretical probabilities is very difficult.

In the Discussion following Example 17, students should conclude that it is unlikely that a student would randomly guess *all* of the answers to a test. If a student states that "I guessed a lot of the answers", this implies that the student did know the answer to some of the questions. We would therefore expect the probability of the student answering at least two questions correctly to be higher than the probability calculated in the worked example.

Many of the questions in this exercise require students to generate random numbers of their own, so students should be aware that their own answers will not exactly match those given in the back of the book. If students are obtaining answers that are vastly different to those in the back of the book, however, they should consider repeating the question with a new set of random numbers. If they still get very different answers to those in the back of the book, their method may be incorrect.

CHAPTER 20: STATISTICS

A Discrete numerical data
B Continuous numerical data
C Describing the distribution of data
D Measures of centre
E Box-and-whisker plots
F Cumulative frequency graphs
G Evaluating reports

Keywords:

- biased sample
- bimodal
- bimodal distribution
- box-and-whisker plot
- box plot
- categorical data
- categorical variable
- census
- class interval
- column graph
- continuous numerical variable
- cumulative frequency
- cumulative frequency graph
- discrete numerical variable
- distribution
- dot plot
- five-number summary
- frequency histogram
- histogram
- interquartile range
- interval midpoint
- lower quartile
- maximum value
- mean
- median
- minimum value
- modal class
- mode
- negatively skewed distribution
- numerical data
- numerical variable
- outlier
- parallel box-and-whisker plot
- percentile
- population
- positively skewed distribution
- range
- sample
- sample size
- statistics
- survey
- symmetric distribution
- tally and frequency table
- upper quartile
- variable

The chapter opens in much the same way as the first edition. However, we have moved the material on "Describing the distribution of data" to its own section to match what we did for Year 9.

In the "Measures of centre" Section, we have combined the raw data and the frequency table subsections so that it matches the structure in the new Year 9 book.

The "Life expectancy" Activity on page 352 is mostly the same as its previous incarnation in the first edition, but includes updated data and columns for Western Australia.

Cumulative frequency graphs has been moved after box-and-whisker plots as we feel that it is better to introduce quartiles first to provide:

- a gentler introduction to percentiles
- more motivation for *using* cumulative frequency graphs.

In the Investigation at the end of Section G, students may need extra guidance choosing a report from the ABS website to read.

CHAPTER 21: BIVARIATE STATISTICS

A Association between categorical variables
B Association between numerical variables
C Correlation
D Line of best fit

Keywords:

- bivariate data
- bivariate statistics
- correlation
- dependent variable
- extrapolation
- independent variable
- interpolation
- linear model
- line of best fit
- mean point
- pole
- scatter plot
- two-way table

The "Line graphs" Section from the first edition has been removed and in its place is the "Association between categorical variables" Section, which is new for Version 9.0 of the Australian Curriculum. This section focuses on data for two categorical variables organised in a two-way table, which, while similar to the two-way tables covered in probability, considers them from the perspective of data and inference.

The "Association between numerical variables" Section is adapted from the "Scatter plots" Section from the first edition. However there is a greater emphasis on their construction and interpretation of the data as a whole.

While Pearson's correlation coefficient and linear regression may not be in the Year 10 standard course, we have included online activities on them on pages 384 and 387 respectively.

CHAPTER 22: RELATIONS AND FUNCTIONS

A Relations and functions
B Function notation
C Domain and range
D Transformations of graphs

Keywords:

- domain
- function
- function notation
- function value
- image
- interval notation
- range
- reflection
- relation
- scale factor
- stretch
- transformation
- translation

This is intended to be a relatively simple chapter, and students will probably find it a welcome change from the previous few chapters. The purpose of the chapter is to put in place the terminology and structures that will be encountered when studying quadratic functions and exponential functions in later chapters.

In this edition, domain and range, which was part of Section A (Relations) in the previous edition, have been placed in their own section to give them more emphasis. Section A (Relations and functions) now focuses on distinguishing between a relation and a function, which occurred in Section B (Functions) in the previous edition.

In Section D, horizontal stretches of functions have been removed in this edition, since they are not used later in the book.

In the Discussion at the end of Section D, students should conclude that:

- No points are invariant when a function is translated, since all points move the same distance in the same direction.
- When a function is stretched vertically or reflected in the x-axis, points on the x-axis are invariant.
- When a function is reflected in the y-axis, points on the y-axis are invariant.

CHAPTER 23: QUADRATIC FUNCTIONS

A Quadratic functions
B Graphs of quadratic functions
C Using transformations to graph quadratics
D Axes intercepts
E Axis of symmetry of a quadratic
F Vertex of a quadratic
G Finding a quadratic function
H Problem solving with quadratic functions

Keywords:

- axis of symmetry
- maximum value
- minimum value
- parabola
- quadratic function
- turning point
- vertex
- x-intercept
- y-intercept

In Section B, we have added an introductory Investigation in which students use the geometric definition to generate a parabola. There is also an Investigation at the end of the section which explores one of the properties of parabolas. Students should find that all of the rays reflected off the parabola pass through a single point, which is the focus of the parabola.

In Section C (Using transformations to graph quadratics), the more involved Investigation which was in the previous edition has been moved to Year 9. Instead, we first establish some basic ideas about the shape and position of $y = (x - h)^2 + k$ and $y = -x^2$ from our work on transformations in the previous chapter. We then investigate the effect of the value of a on the shape of the graph, and the direction in which it opens.

In Question **7** of Exercise 23C, students must convert the quadratic functions into completed square form. In Questions **8** to **10**, students verify the given completed square form for quadratic functions with $a \neq 1$.

Section G (Finding a quadratic function) has been added in this edition. Finding a function based on information about its graph is an important modelling skill which students may find useful in later years.

The "Quadratic optimisation" Section has been renamed "Problem solving with quadratic functions" (Section H), since students are asked to solve problems other than identifying the optimum point.

CHAPTER 24: EXPONENTIALS AND LOGARITHMS

A Exponential functions
B Graphs of exponential functions
C Exponential equations
D Exponential growth
E Exponential decay
F Logarithms
G Logarithm of a product
H Logarithmic scales

Keywords:

- common logarithm
- exponential decay
- exponential equation
- exponential function
- exponential growth
- horizontal asymptote
- logarithm
- logarithmic scale
- order of magnitude
- y-intercept

This will be the first time students have encountered exponential functions. Students should understand that, while linear functions are characterised by a quantity changing by a constant *amount* each time period, exponential functions are characterised by a quantity changing by a constant *percentage* each time period.

In this edition, we have added a section on solving exponential equations (Section C), since it is now included in the syllabus. Students first solve exponential equations algebraically by writing each side of the equation with the same base, and then equating the exponents. Students then solve some more complicated exponential equations using technology. This can either be done graphically, or by using the Solver function of the calculator. This can then be used in exponential growth and decay to answer questions such as "How long will it take for the population to reach 500?".

Logarithms have also been added in this edition. In Section F, students are introduced to the concept of a logarithm, and hopefully get a feel for how logarithms work.

Section G (Logarithm of a product) is not explicitly part of the syllabus, but the rule $\log(ab) = \log a + \log b$ is useful in the understanding of logarithmic scales, which appear in the following section.

Students should be familiar with some of the logarithmic scales mentioned in Section H, such as the Richter scale. Students should be encouraged to solve these problems by thinking them through intuitively, rather than by algebraic means. For example, in the Richter scale, each increase of 1 in the magnitude means that the intensity is multiplied by 10. If students understand this, then it is fairly straightforward to deduce that, for example, a magnitude 4 earthquake is $10 \times 10 = 100$ times more intense than a magnitude 2 earthquake, and that an earthquake that is 10 times the intensity of a magnitude 4 earthquake has magnitude $4 + 1 = 5$. Trying to solve these problems using the formula $M = \log\left(\frac{I}{I_0}\right)$ is likely to lead to confusion.