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### YEAR 12 MATHEMATICS SPECIALIST

#### CHAPTER 1: MATHEMATICAL INDUCTION

	WACE syllabus reference
A The process of induction	} SACE only
B The principle of mathematical induction	

Induction is in the SACE syllabus, but not the WACE syllabus at Year 12. Thus, Western Australian students should omit this chapter.

#### CHAPTER 2: REAL POLYNOMIALS

A Operations with polynomials	
B Zeros, roots, and factors	
C Polynomial equality	
D Polynomial division	
E The Remainder theorem	3.1.13
F The Factor theorem	3.1.13
G The Fundamental Theorem of Algebra	3.1.14
H Sum and product of roots theorem (Extension)	
I Graphing real polynomials	
J Polynomial equations	3.1.15

In this chapter we consider the properties, operations, theorems, and graphs associated with real polynomials.

Polynomial division by a linear factor is presented using both long division and synthetic division. The long division method is presented first, and students get practice with this method, as they will need it for when they divide by quadratics. We then present synthetic division, so that students can divide by linear factors faster.

#### CHAPTER 3: FUNCTIONS

A Composite functions	3.2.1, 3.2.2
B Inverse functions	3.2.3, 3.2.4, 3.2.5
C Reciprocal functions	
D The reciprocal of other functions	3.2.7
E Rational functions	3.2.8
F Absolute value functions	3.2.6, 3.2.7

This Topic 2 chapter has been presented before the remainder of Topic 1. This was done so that the absolute value function and properties of modulus are explained in a real number context before we study the modulus of complex numbers.

At the end of Section B there is an Investigation on the inverse trigonometric functions. Students should be encouraged to complete this Investigation as these functions appear in the vector and integration chapters later in the book.

#### **CHAPTER 4: COMPLEX NUMBERS**

A	The complex plane	3.1.1, 3.1.2, 3.1.3, 3.1.8, 3.1.9
B	Modulus and argument	3.1.4, 3.1.10
C	Polar form	3.1.5, 3.1.6
D	Euler's form	
E	De Moivre's theorem	3.1.7
F	Roots of complex numbers	3.1.11, 3.1.12

Sections A and B are largely revision of what was done in Year 11, so these sections should be worked through swiftly. An Activity on the triangle inequality is included at the end of Section B, however this was also addressed in Year 11, and students who completed the work in Year 11 need not complete the Activity.

The remaining sections extend what was done in Year 11 to consider polar and Euler forms of complex numbers. Just as we use the Cartesian form of a complex number to represent addition and subtraction of complex numbers on an Argand plane, students should understand the power of the polar form to represent the multiplication of complex numbers. This will help students appreciate the use of polar form in finding powers and roots of complex numbers.

#### **CHAPTER 5: VECTORS**

A	Vectors in space	3.3.1, 3.3.3
B	Operations with vectors in space	3.3.1
C	Vector algebra	
D	The vector between two points	
E	Parallelism	3.3.2, 3.3.3
F	The scalar product of two vectors	
G	The angle between two vectors	
H	Proof using vector geometry	3.3.2
I	The vector product of two vectors	3.3.7

In Year 11, students explored vectors in two dimensions. In Year 12, the focus moves to three dimensional vectors. With the extra dimension, it becomes more difficult to visualise the vectors on the page. This makes it all the more important that students can operate with vectors algebraically.

## CHAPTER 6: VECTOR APPLICATIONS

A	Area	
B	Lines in 2 and 3 dimensions	3.3.4, 3.3.5
C	The angle between two lines	
D	Constant velocity problems	3.3.5, 3.3.6
E	The shortest distance from a point to a line	
F	Intersecting lines	3.3.6
G	Relationships between lines	3.3.6
H	Planes	3.3.7, 3.3.8
I	Angles in space	
J	Solving $3 \times 3$ linear systems	3.3.9
K	Intersecting planes	3.3.9, 3.3.10

We now look at some applications of vectors.

In Section B we consider the equations of lines in 2 and 3 dimensions. Instead of presenting the 2-dimensional and 3-dimensional cases separately, we simply give the equation for the 3-dimensional case, and instruct students to ignore the  $z$ -coordinate for the 2-dimensional case.

We begin the study of planes with an Investigation showing that, given two non-parallel vectors in the plane, we can reach any point on the plane by taking a linear combination of these vectors. This helps to motivate the equation of the plane.

## CHAPTER 7: INTEGRATION

A	Rules for integration	4.1.1, 4.1.3, 4.1.4
B	Integrating $\pm \frac{1}{\sqrt{a^2 - x^2}}$ and $\frac{a}{a^2 + x^2}$	
C	Integration by substitution	4.1.2
D	Integration by parts	
E	The area between two functions	4.1.5
F	Solids of revolution	4.1.6, 4.1.7

This chapter follows directly from the integration studied in the Mathematics Methods course. For this reason, it is important that the Mathematics Methods integration chapters are completed before this chapter is started.

In Section B, we differentiate the inverse trigonometric functions. This allows us to integrate expressions of the form  $\pm \frac{1}{\sqrt{a^2 - x^2}}$  and  $\frac{a}{a^2 + x^2}$ . It will be beneficial for

students to complete the Investigation on inverse trigonometric functions in Chapter 3, so they are familiar with the functions upon reaching this section.

## CHAPTER 8: RATES OF CHANGE AND DIFFERENTIAL EQUATIONS

A	Implicit differentiation	4.2.1
B	Related rates	4.2.2
C	Differential equations	
D	Differential equations of the form $\frac{dy}{dx} = f(x)$	4.2.4
E	Separable differential equations	4.2.4, 4.2.6
F	Slope fields	4.2.5
G	Problem solving	4.2.4, 4.2.6
H	Logistic growth	4.2.6
I	Equations of motion	4.2.7
J	Simple harmonic motion	4.2.7

In this chapter, we extend the differentiation work done in Mathematics Methods, and look at implicit differentiation, related rates, and solving differential equations.

This chapter is quite long, so teachers should make sure to allocate plenty of time to complete it.

## CHAPTER 9: VECTOR CALCULUS

A	Parametric equations	3.3.11, 3.3.12
B	Pairs of uniformly varying quantities	3.3.13, 3.3.14
C	Pairs of non-uniformly varying quantities	3.3.13, 3.3.15
D	Bézier curves	
E	Trigonometric parameterisation	3.3.12, 3.3.15
F	Arc lengths of parametric curves	

We complete our study of calculus by considering motion represented as parametric curves.

Sections D and F cover material explicitly mentioned in the SACE syllabus only, so Western Australian students can omit these sections.