HAESE MATHEMATICS
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TEACHER NOTES FOR YEAR 12 SPECIALIST MATHEMATICS

CHAPTER 1: MATHEMATICAL INDUCTION

		SACE	ACARA
A	The process of induction	Topic 1	
B	The principle of mathematical induction	Sub-topic 1.1	

Induction is in the SACE syllabus, but not the ACARA syllabus. Thus, only South Australian students need to complete this chapter.

This chapter builds on the induction work done in Chapter 6 of the Year 11 textbook.

Induction was only added to the SACE Stage 2 syllabus at the final draft stage, after our Year 11 induction chapter had been completed. The induction work required in the Year 12 syllabus was in fact very similar to what was already in our Year 11 book. Therefore, if students did not complete all of the problems in the Year 11 induction chapter, these problems would be suitable for extra revision.

Problems involving proof by induction are also included throughout the book where appropriate.

CHAPTER 2: REAL POLYNOMIALS

		SACE	ACARA
A	Operations with polynomials		
B	Zeros, roots, and factors		
C	Polynomial equality		
D	Polynomial division		
E	The remainder theorem	Topic 2	Unit 3
F	The factor theorem	Sub-topic 2.4	Topic 1
G	The Fundamental Theorem of Algebra		
H	Sum and product of roots theorem (Extension)		
I	Graphing real polynomials		
J	Polynomial equations		

We begin our study of Topic 2 by considering the properties, operations, theorems, and graphs associated with real polynomials.

Polynomial division by a linear factor is presented using both long division and synthetic division. The long division method is presented first, and students get practice with this method, as they will need it for when they divide by quadratics. We then present synthetic division, so that students can divide by linear factors faster.

CHAPTER 3: FUNCTIONS

		SACE	ACARA
A	Composite functions	Topic 3	
		Sub-topic 3.1	
B	Inverse functions	Sub-topic 3.2	Unit 3
C	Reciprocal functions		Topic 2
D	The reciprocals of other functions	Sub-topic 3.3	
E	Rational functions		
F	Absolute value functions		

This SACE Topic 3 chapter has been presented before the remainder of Topic 2. This was done so that the absolute value function and properties of modulus are explained in a real number context before we study the modulus of complex numbers.

At the end of Section B there is an Investigation on the inverse trigonometric functions. Students should be encouraged to complete this Investigation as these functions appear in the vector and integration chapters later in the book.

CHAPTER 4: COMPLEX NUMBERS

		SACE	ACARA
A	The complex plane		
B	Modulus and argument	Topic 2	
C	Polar form	Sub-topics 2.1,	Unit 3
D	Euler's form	2.2	Topic 1
E	De Moivre's theorem		
F	Roots of complex numbers	Sub-topic 2.3	

Sections A and B are largely revision of what was done in Year 11, so these sections should be worked through swiftly. An Activity on the triangle inequality is included at the end of Section B, however this was also addressed in Year 11, and students who completed the work in Year 11 need not complete the Activity.

The remaining sections extend what was done in Year 11 to consider polar and Euler forms of complex numbers. Just as we use the Cartesian form of a complex number to represent addition and subtraction of complex numbers on an Argand plane, students should understand the power of the polar form to represent the multiplication of complex numbers. This will help students appreciate the use of polar form in finding powers and roots of complex numbers.

CHAPTER 5: VECTORS

		SACE	ACARA
A	Vectors in space	Topic 4 Sub-topic 4.1	
B	Operations with vectors in space		
C	Vector algebra		Unit 3
D	The vector between two points		Topic 3
E	Parallelism	Sub-topic 4.2	
F	The scalar product of two vectors		
G	The angle between two vectors		
H	Proof using vector geometry		
I	The vector product of two vectors		

In Year 11, students explored vectors in two dimensions. In Year 12, the focus moves to three dimensional vectors. With the extra dimension, it becomes more difficult to visualise the vectors on the page. This makes it all the more important that students can operate with vectors algebraically.

CHAPTER 6: VECTOR APPLICATIONS

		SACE	ACARA
A	Area		
B	Lines in 2 and 3 dimensions		
C	The angle between two lines		
D	Constant velocity problems		
E	The shortest distance from a point to a line	Topic 4 Sub-topic 4.2	Unit 3 Topic 3
F	Intersecting lines		
G	Relationships between lines		
H	Planes		
I	Angles in space		
J	Solving 3×3 linear systems	Sub-topic 4.3	
K	Intersecting planes		

We complete our work on vectors by looking at some applications of vectors.

In Section B we consider the equations of lines in 2 and 3 dimensions. Instead of presenting the 2-dimensional and 3-dimensional cases separately, we simply give the equation for the 3-dimensional case, and instruct students to ignore the *z*-coordinate for the 2-dimensional case.

We begin the study of planes with an Investigation showing that, given two non-parallel vectors in the plane, we can reach any point on the plane by taking a linear combination of these vectors. This helps to motivate the equation of the plane.

CHAPTER 7: INTEGRATION

		SACE	ACARA
A	Rules for integration	Topic 5 Sub-topic 5.1	Unit 4 Topic 1
B	Integrating $\pm\frac{1}{\sqrt{a^2-x^2}}$ and $\frac{a}{a^2+x^2}$		
C	Integration by substitution		
D	Integration by parts		
E	The area between two functions	Sub-topic 5.2	
F	Solids of revolution		

This chapter follows directly from the integration studied in the Mathematical Methods course. For this reason, it is important that the Mathematical Methods integration chapters are completed before this chapter is started.

In Section B, we differentiate the inverse trigonometric functions. This allows us to integrate expressions of the form $\pm\frac{1}{\sqrt{a^2-x^2}}$ and $\frac{a}{a^2+x^2}$. It will be beneficial for students to complete the Investigation on inverse trigonometric functions in Chapter 3, so they are familiar with the functions upon reaching this section.

CHAPTER 8: RATES OF CHANGE AND DIFFERENTIAL EQUATIONS

		SACE	ACARA
A	Implicit differentiation	Topic 6 Sub-topic 6.1	
B	Related rates	Sub-topic 6.2	Unit 4 Topic 2
C	Differential equations		
D	Differential equations of the form $\frac{dy}{dx}=f(x)$		
E	Separable differential equations		
F	Slope fields		
G	Problem solving		
H	Logistic growth		
I	Equations of motion		
J	Simple harmonic motion		

In this chapter, we extend the differentiation work done in Mathematical Methods, and look at implicit differentiation, related rates, and solving differential equations.

This chapter is quite long, so teachers should make sure to allocate plenty of time to complete it.

Sections I and J are part of the ACARA syllabus, but not the SACE syllabus, so South Australian students need not study these sections.

CHAPTER 9: VECTOR CALCULUS

		SACE	ACARA
A	Parametric equations	Topic 6 Sub-topic 6.3	Unit 3 Topic 3
B	Pairs of uniformly varying quantities		
C	Pairs of non-uniformly varying quantities		
D	Bézier curves		
E	Trigonometric parameterisation	Sub-topic 6.5	Unit 3 Topic 3
F	Arc lengths of parametric curves	Sub-topic 6.4	

We complete our study of calculus by considering motion represented as parametric curves. For South Australian teachers, this material is similar to what was in the Calculus chapter of the old Specialist Mathematics course. However, the final section, regarding the arc length of parametric curves, was not in the old Specialist Mathematics course, and so is likely to be unfamiliar to many teachers.

Students following the ACARA syllabus need not complete Sections D and F, as these sections are part of the SACE syllabus only.