MATHEMATICS FOR THE INTERNATIONAL STUDENT MATHEMATICAL SL EXAM PREPARATION AND PRACTICE GUIDE (as at 10 May 2007)

page 9 **PRACTICE QUESTIONS (Topic 1)**

7 b Graph caption should be: $y = -x^2 - 2$

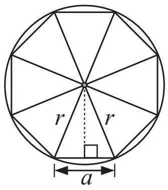
page 44 **TRIAL EXAMINATION 3**

1 a Joise should have bought two books by Catherine Cookson.

page 45 **TRIAL EXAMINATION 4**

5 Question should be:

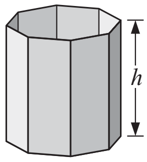
A circle of radius r encloses a regular octagon as shown.



a Show that a and r are related by $a = 2r\sin(\frac{\pi}{8})$.

b Show that the area of the octagon is $8r^2\sin(\frac{\pi}{8})\cos(\frac{\pi}{8})$.

c



Octagonal waste paper bins are to be made of fixed volume V with a variable radius r (as shown in the first diagram) and a height h.

i Show that the area of material needed for the base and sides is given by A where

$A = 8r(2h + r\cos(\frac{\pi}{8}))\sin(\frac{\pi}{8})$ and that

$V = 8\left[\sin(\frac{\pi}{8})\cos(\frac{\pi}{8})\right]r^2h$.

ii Hence, show that $A = \dfrac{2V}{r\cos(\frac{\pi}{8})} + 8r^2\sin(\frac{\pi}{8})\cos(\frac{\pi}{8})$.

iii Show that A is a minimum when

$$r = \left(\frac{V}{8\sin(\frac{\pi}{8})\cos^2(\frac{\pi}{8})}\right)^{\frac{1}{3}}.$$

page 51 **SOLUTIONS TO TOPIC 1**

21 c Pierre had $\$10 \times 52 + \$100 = \$620$

Francesca had $(0.50 + 1 + 1.50 + \ldots\ldots + 26) + 100$

$= \frac{26}{2}(0.5 + 26) + 100$

$= 13 \times 26.5 + 100$

$= \$444.50$

page 53 **SOLUTIONS TO TOPIC 2**

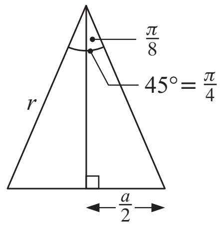
15 d Any line parallel to the y-axis is of the form $x = k$

$\therefore$ equation is $x = 4$.

page 116 **SOLUTIONS TO TRIAL EXAMINATION 4**

5 Solution should be:

a



$\sin\frac{\pi}{8} = \dfrac{\frac{a}{2}}{r}$

$\therefore \dfrac{a}{2} = r\sin\frac{\pi}{8}$

$\therefore a = 2r\sin\frac{\pi}{8}$

b Area of octagon $= 8 \times$ area of one triangle

$= 8 \times \frac{1}{2}r^2\sin\frac{\pi}{4}$

$= 4r^2(2\sin\frac{\pi}{8}\cos\frac{\pi}{8})$

$= 8r^2\sin\frac{\pi}{8}\cos\frac{\pi}{8}$

c i $A =$ area of base $+$ area of sides

$= 8r^2\sin\frac{\pi}{8}\cos\frac{\pi}{8} + 8 \times a \times h$

$= 8r^2\sin\frac{\pi}{8}\cos\frac{\pi}{8} + 8(2r\sin\frac{\pi}{8})h$

$\therefore A = 8r(r\cos\frac{\pi}{8} + 2h)\sin\frac{\pi}{8}$ .... (1)

and $V =$ area of base $\times$ height

$= 8r^2\sin\frac{\pi}{8}\cos\frac{\pi}{8} \times h$

$\therefore V = 8r^2h\sin\frac{\pi}{8}\cos\frac{\pi}{8}$ .... (2)

ii From (2), $h = \dfrac{V}{8r^2\sin\frac{\pi}{8}\cos\frac{\pi}{8}}$

Substituting into (1),

$$A = 8r\sin\frac{\pi}{8}\left(r\cos\frac{\pi}{8} + \frac{V}{4r^2\sin\frac{\pi}{8}\cos\frac{\pi}{8}}\right)$$

$$\therefore A = 8r^2\sin\frac{\pi}{8}\cos\frac{\pi}{8} + \frac{2V}{r\cos\frac{\pi}{8}}$$

iii As $A = 8r^2\sin\frac{\pi}{8}\cos\frac{\pi}{8} + \dfrac{2Vr^{-1}}{\cos\frac{\pi}{8}}$

$\therefore \dfrac{dA}{dr} = 16r\sin\frac{\pi}{8}\cos\frac{\pi}{8} - \dfrac{2Vr^{-2}}{\cos\frac{\pi}{8}}$

which is 0 when $16r\sin\frac{\pi}{8}\cos\frac{\pi}{8} = \dfrac{2V}{r^2\cos\frac{\pi}{8}}$

$\therefore r^3 = \dfrac{2V}{16\sin\frac{\pi}{8}\cos^2\frac{\pi}{8}}$

$\therefore r = \left(\dfrac{V}{8\sin\frac{\pi}{8}\cos^2\frac{\pi}{8}}\right)^{\frac{1}{3}}$

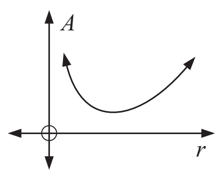
2nd derivative test

$\dfrac{d^2A}{dr^2} = 16\sin\frac{\pi}{8}\cos\frac{\pi}{8} + \dfrac{4Vr^{-3}}{\cos\frac{\pi}{8}}$

$= 16\sin\frac{\pi}{8}\cos\frac{\pi}{8} + \dfrac{4V}{r^3\cos\frac{\pi}{8}}$

> 0 for all $r > 0$

$\therefore$ shape of graph is



$\therefore$ min. value A occurs when $r = \left(\dfrac{V}{8\sin\frac{\pi}{8}\cos^2\frac{\pi}{8}}\right)^{\frac{1}{3}}$.