

ERRATA

MATHEMATICS FOR THE INTERNATIONAL STUDENT MATHEMATICS HL (OPTIONS)

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page 31 **TABLE** replace row 2

Binomial	$X \sim B(n, p)$	$\binom{n}{x} p^x (1-p)^{n-x}$ for $x = 0, 1, \dots, n$	np	$np(1-p)$
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page 54 **EXAMPLE 30** solution, second to last line should read:

$$= \frac{1}{n^2} \times n\sigma^2$$

page 91 **EXAMPLE 51** solution, lines 3 to 5 should read:

$$\begin{aligned} P(X = 4) &= \text{poissonpdf}(5, 4) \approx 0.1755 && \text{and } 36 \times 0.1755 \approx 6.32 \\ P(X = 5) &= \text{poissonpdf}(5, 5) \approx 0.1755 && \text{and } 36 \times 0.1755 \approx 6.32 \\ P(X = 6) &= \text{poissonpdf}(5, 6) \approx 0.1462 && \text{and } 36 \times 0.1462 \approx 5.26 \end{aligned}$$

page 93 **EXAMPLE 53** solution, last 6 lines on the page should read:

$$\begin{aligned} P(X = 0) &= \text{binompdf}(5, 0.5346, 0) \approx 0.02183 && \text{and } 150 \times 0.02183 \approx 3.3 \\ P(X = 1) &= \text{binompdf}(5, 0.5346, 1) \approx 0.12540 && \text{and } 150 \times 0.12540 \approx 18.8 \\ P(X = 2) &= \text{binompdf}(5, 0.5346, 2) \approx 0.28810 && \text{and } 150 \times 0.28810 \approx 43.2 \\ P(X = 3) &= \text{binompdf}(5, 0.5346, 3) \approx 0.33093 && \text{and } 150 \times 0.33093 \approx 49.6 \\ P(X = 4) &= \text{binompdf}(5, 0.5346, 4) \approx 0.19007 && \text{and } 150 \times 0.19007 \approx 28.5 \\ P(X = 5) &= \text{binompdf}(5, 0.5346, 5) \approx 0.04367 && \text{and } 150 \times 0.04367 \approx 6.6 \end{aligned}$$

page 94 **EXAMPLE 53** solution, replace the table, “**Test Statistic**” and “**p-value**”:

<i>Number of children</i>	0 or 1	2	3	4	5	Σ
f_o	23	41	52	26	8	150
f_e	22.1	43.2	49.6	28.5	6.6	150

Test Statistic: $\chi_{calc}^2 = \sum \frac{(f_o - f_e)^2}{f_e} \approx 0.806$ {graphics calculator}

p-value: $p\text{-value} = P(\chi_{calc}^2 > 0.806) \approx 0.848$ {graphics calculator}

page 95 **EXAMPLE 54** solution, change the following line to:

p-value: $p\text{-value} = P(\chi_{calc}^2 > 10.696) \approx 0.0135$ (from the gcalc.)

page 107 **REVIEW SET 8B**

- 12** A drink manufacturer produces soft drink for sale with each bottle having contents advertised at 375 mL. It is known that the machines producing these drinks are set so that the average volume per bottle produced is 376 mL with a standard deviation of 1.84 mL. Given that the volumes of bottles are distributed normally, find:

page 184 **EXAMPLE 10** solution, second to last line should read:

If $p = 1$ then we have the case presented in **Example 9**, which is divergent.

page 193 **TEXT – The Squeeze Theorem** the last line of the proof should read:

Hence $\lim_{n \rightarrow \infty} b_n = L.$

$$\text{where } R_n(x : a) = \frac{f^{(n+1)}(c)(x - a)^{n+1}}{(n + 1)!}, \text{ where } c \text{ is a constant, } c \text{ between } x \text{ and } a$$

10 b Prove that the square of an integer is of the form $4q$ or $4q + 1$ for some $q \in \mathbb{Z}$.

14 Using the result of the previous question, show that the fourth power of any odd integer is of the form $8k + 1$.

Proof should reference Theorem 4, not Theorem 1.

Solve $172x + 20y = 1000$ in: **a** \mathbb{Z} **b** \mathbb{Z}^+

- A final type are those of the form $2^{2^n} + 1$; these are the Fermat primes. $n = 3, 5$

Thus, $a(2a)(3a)(4a)\dots(p - 1)a \equiv (1)(2)(3)(4)\dots(p - 1) \pmod{p}$

$$\therefore a^{p-1}(p - 1)! \equiv (p - 1)! \pmod{p}$$

Now since $p \nmid (p - 1)!$, p being prime, we can cancel by $(p - 1)!$

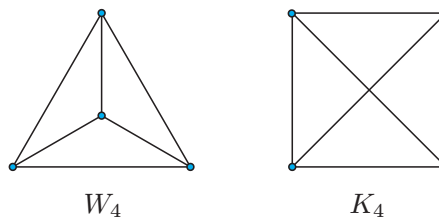
$$\therefore a^{p-1} \equiv 1 \pmod{p}$$

- Graph **5** is both W_4 and K_4 .

Incident Edge/Vertex An edge which connects two adjacent vertices is said to be incident on each vertex.

Isomorphism is an important concept in many areas of mathematics. You may have met it in other areas of the IB Higher Level Mathematics course, such as in Group Theory.

In **Section 11B.2**, we briefly introduced graph isomorphism when we compared the wheel graph W_4 with the complete graph K_4 . We saw that these graphs have seemingly different representations on paper, as illustrated alongside, but they are in fact the same. Here they are again (and there are many other representations):



A connected graph is **Eulerian** if and only if all of its vertices are even.

A connected graph is traversable if and only if at most two of its vertices are odd.

- 4** $M \sim N(61, 11^2)$ and $C \sim N(48, 4^2)$
 $U = M_1 + M_2 + M_3 + M_4 + C_1 + C_2 + C_3$
 $U \sim N(388, 532)$
 $P(U > 440) \approx 0.0121$ if unsafe
Assumption: The random variables $M_1, M_2, M_3, M_4,$
 C_1, C_2 and C_3 are independent.

- 4 b** t -distribution with $s_{n-1} = \sqrt{\frac{389}{388}} \times \0.25
 ≈ 0.2503

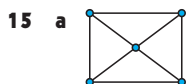
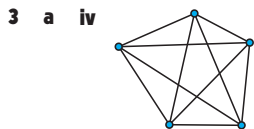
5 c ii F is now binomial
 i.e., $F \sim B(12, \frac{1}{4})$ and
 $P(\text{buy packet})$
 $= 1 - P(\text{do not buy packet})$
 $= 1 - \{P(F = 2) + P(F = 1) \times \frac{1}{4}\}$
 $= 1 - \left\{ \binom{2}{2} \left(\frac{1}{4}\right)^2 + \binom{2}{1} \left(\frac{1}{4}\right)^1 \left(\frac{3}{4}\right)^1 \times \frac{1}{4} \right\}$
 ≈ 0.844

3 c $\sum_{i=0}^{\infty} e^{-(i+1)^2} < \int_0^{\infty} e^{-x^2} dx < \sum_{i=0}^{\infty} e^{-i^2}$
4 c $\sum_{i=1}^{\infty} \frac{1}{(i+1)^2} < \int_1^{\infty} \frac{1}{x^2} dx < \sum_{i=1}^{\infty} \frac{1}{i^2}$

1 $\ln(1+x)$
 $= x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \dots + (-1)^{n-1} \frac{x^n}{n} + R_n(x:0)$
 Interval of convergence $] -1, 1]$.
 $R_n(x:0) = f^{(n+1)}(c) \frac{(x-0)^{n+1}}{(n+1)!}$
 $= \frac{n!(-1)^n}{(1+c)^{n+1}} \frac{x^{n+1}}{(n+1)!}$
 $= \frac{(-1)^n}{(1+c)^{n+1}(n+1)} x^{n+1}$
 $\rightarrow 0$ for $|x| < 1, -1 < c < 1$

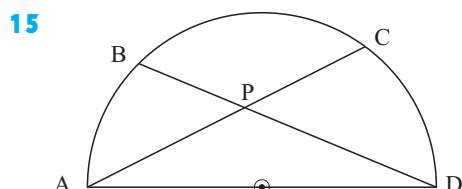
14 The square of an odd number has form $4p+1, p \in \mathbb{Z}$
 (Question **13c**)
 i.e., $a^2 = 4p+1$
 $\Rightarrow a^4 = (4p+1)^2 = 16p^2 + 8p + 1$
 $\Rightarrow a^4 = 8(2p^2 + p) + 1$
 which is of the form $8k+1, k \in \mathbb{Z}$

1 c iii 2, 2, 4, 4



- 10** Triangle ABC has altitudes [AP] and [BQ] where P lies on [BC] and Q lies on [AC]. H is the intersection of [AP] and [BQ].
 Prove that $AH \cdot HP = BH \cdot HQ$.

- 8** [XAB] and [XC] are two intersecting straight line segments.
 Given that $BX = 6.4$ m, $AB = 5.5$ m and $XC = 2.4$ m, prove that [CX] is a tangent to the circle through A, B and C.



ABCD is a semi-circle with a diameter [AD].
 P is the point of intersection of [AC] and [BD].
 Prove that:

$$AP \cdot AC + DP \cdot DB = AD^2$$

- 9** For $(x - 2)^2 + (y - 2)^2 = 4$ and $x^2 + y^2 = 4$:
11 a Find the equations of the radical axes of the circles $(x + 2)^2 + (y + 4)^2 = 17$ and $(x - 1)^2 + (y - 5)^2 = 5$. Do these circles intersect?

(Remove question **12**)

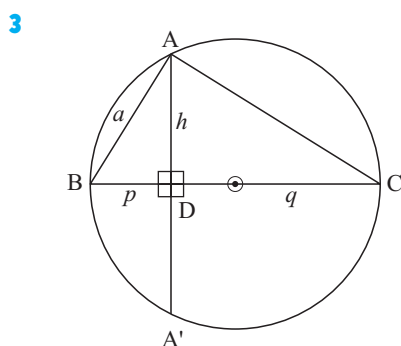
- 7** P is the midpoint of [BC] of triangle ABC. [PQ] is the bisector of angle APB and cuts AB at Q. [QR] is drawn parallel to [BC], meeting [AC] at R. Prove that angle QPR is a right angle.

- 9** [AB] is a fixed diameter of a circle and F is a fixed point on [AB]. P and Q are points on the circle such that [PQ] is parallel to [AB] and is variable. Prove that $FP^2 + FQ^2$ is constant.

- 4 b** If the other diagonal has length y units, deduce that $y^2 = \frac{(ac + bd)(ad + bc)}{(ab + cd)}$

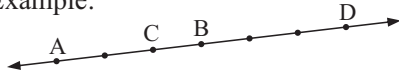
- 6** Similarly to **5**, use Ptolemy's theorem and the figure alongside to prove that $\sin(\alpha - \beta) = \sin\alpha \cos\beta - \cos\alpha \sin\beta$.

- 2** Notice that Euclid's theorem (in a right angled triangle) was proved using similar triangles only. Use Euclid's theorem to prove that $a^2 + b^2 = (p + q)^2$, i.e., to prove Pythagoras' theorem.



- a** [BC] is the diameter of a circle, centre O. A lies on the circle and A' is the image of A under a reflection in the line [BC]. Use the 'intersecting chords' theorem to deduce that $h^2 = pq$.
b By drawing another circle on the diagram in **a**, prove that $a^2 = p(p + q)$ using the 'intersecting chords theorem - special case'.

Example:



$AC : CB = 2 : 1$
and $AD : DB = 6 : 3$ or $2 : 1$.

So, C and D divide [AB] harmonically
and the harmonic ratio is $2 : 1$.

1



M divides [PQ] internally in the ratio $1 : 3$.

Locate N such that M and N divide:

- a [PQ] harmonically
- b [QP] harmonically. Illustrate.

1 (remove part b)

2 P, Q and R lie on sides [AB], [BC] and [CA] of triangle ABC.

If $AP = \frac{2}{3}AB$, $BQ = \frac{3}{4}BC$ and $CR = \frac{1}{7}CA$, prove that [AQ], [BR] and [CP] are concurrent.

2 d $r = \sqrt{31}$ cm

4 c $OX = \sqrt{105}$ cm

10 a (1, 3) and (3, -3) b $y = 6 - 3x$, $1 \leq x \leq 3$

c $y = 6 - 3x$

d $y = 6 - 3x$, the equation of the radical axis

11 a $x + 3y = 3$ No, they do not intersect.

3 $\frac{\sqrt{208}}{3}$ cm, $\frac{10}{3}$ cm, $\frac{\sqrt{292}}{3}$ cm

3 $\sqrt{105}$ cm

1 b $\frac{144}{13}$ cm c $\frac{60}{13}$ cm

2 b $16 : 33$ c $\frac{98}{11}$ cm²

1 b $1 : 2$

2 $\frac{AP}{PB} \times \frac{BQ}{QC} \times \frac{CR}{RA} = \frac{2}{1} \times \frac{3}{1} \times \frac{1}{6} = 1$ etc.

4 $1 : 2$