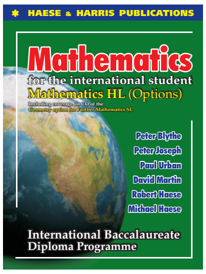


ERRATA
MATHEMATICS FOR THE INTERNATIONAL STUDENT
MATHEMATICS HL (OPTIONS)

First edition - 2005 initial print run

page 31 **TABLE** replace row 2

Binomial	$X \sim \mathrm{B}(n, p)$	$\binom{n}{x} p^x(1-p)^{n-x}$ for $x = 0, 1, ..., n$	np	$np(1-p)$

page 54 **EXAMPLE 30** solution, second to last line should read:

$$= \frac{1}{n^2} \times n\sigma^2$$

page 91 **EXAMPLE 51** solution, lines 3 to 5 should read:

$\mathrm{P}(X = 4) = \text{poissonpdf}(5, 4) \approx 0.1755$ and $36 \times 0.1755 \approx 6.32$
$\mathrm{P}(X = 5) = \text{poissonpdf}(5, 5) \approx 0.1755$ and $36 \times 0.1755 \approx 6.32$
$\mathrm{P}(X = 6) = \text{poissonpdf}(5, 6) \approx 0.1462$ and $36 \times 0.1462 \approx 5.26$

page 92 **EXAMPLE 52** solution, change the following 6 lines:

$\mathrm{P}(X = 0) = \text{binompdf}(5, 0.5, 0) \approx 0.03125$ and $150 \times 0.03125 \approx 4.7$
$\mathrm{P}(X = 1) = \text{binompdf}(5, 0.5, 1) \approx 0.15625$ and $150 \times 0.15625 \approx 23.4$
$\mathrm{P}(X = 2) = \text{binompdf}(5, 0.5, 2) \approx 0.3125$ and $150 \times 0.3125 \approx 46.9$
$\mathrm{P}(X = 3) = \text{binompdf}(5, 0.5, 3) \approx 0.3125$ and $150 \times 0.3125 \approx 46.9$
$\mathrm{P}(X = 4) = \text{binompdf}(5, 0.5, 4) \approx 0.15625$ and $150 \times 0.15625 \approx 23.4$
$\mathrm{P}(X = 5) = \text{binompdf}(5, 0.5, 5) \approx 0.03125$ and $150 \times 0.03125 \approx 4.7$

page 93 **EXAMPLE 53** solution, last 6 lines on the page should read:

$\mathrm{P}(X = 0) = \text{binompdf}(5, 0.5346, 0) \approx 0.021\,83$ and $150 \times 0.021\,83 \approx 3.3$
$\mathrm{P}(X = 1) = \text{binompdf}(5, 0.5346, 1) \approx 0.125\,40$ and $150 \times 0.125\,40 \approx 18.8$
$\mathrm{P}(X = 2) = \text{binompdf}(5, 0.5346, 2) \approx 0.288\,10$ and $150 \times 0.288\,10 \approx 43.2$
$\mathrm{P}(X = 3) = \text{binompdf}(5, 0.5346, 3) \approx 0.330\,93$ and $150 \times 0.330\,93 \approx 49.6$
$\mathrm{P}(X = 4) = \text{binompdf}(5, 0.5346, 4) \approx 0.190\,07$ and $150 \times 0.190\,07 \approx 28.5$
$\mathrm{P}(X = 5) = \text{binompdf}(5, 0.5346, 5) \approx 0.043\,67$ and $150 \times 0.043\,67 \approx 6.6$

page 94 **EXAMPLE 53** solution, replace the table, "**Test Statistic**" and "***p*-value**":

Number of children	0 or 1	2	3	4	5	$\sum$
f_o	23	41	52	26	8	150
f_e	22.1	43.2	49.6	28.5	6.6	150

Test Statistic: $\chi^2_{calc} = \sum \frac{(f_o - f_e)^2}{f_e} \approx 0.806$ {graphics calculator}

***p*-value:** p-value $= \mathrm{P}(\chi^2_{calc} > 0.806) \approx 0.848$ {graphics calculator}

page 95 **EXAMPLE 54** solution, change the following line to:

***p*-value:** p-value $= \mathrm{P}(\chi^2_{calc} > 10.696) \approx 0.0135$ (from the gcalc.)

page 107 **REVIEW SET 8B**

12 A drink manufacturer produces soft drink for sale with each bottle having contents advertised at 375 mL. It is known that the machines producing these drinks are set so that the average volume per bottle produced is 376 mL with a standard deviation of 1.84 mL. Given that the volumes of bottles are distributed normally, find:

page 184 **EXAMPLE 10** solution, second to last line should read:

If $p = 1$ then we have the case presented in **Example 9**, which is divergent.

page 193 **TEXT – The Squeeze Theorem** the last line of the proof should read:

Hence $\lim_{n \to \infty} b_n = L$.

page 225 **TEXT – Taylor's Theorem** second to last line of the theorem should read:

where $R_n(x : a) = \dfrac{f^{(n+1)}(c)(x-a)^{n+1}}{(n+1)!}$, where c is a constant, c between x and a

page 259 **EXERCISE 11A.3.2**

10 b Prove that the square of an integer is of the form $4q$ or $4q+1$ for some $q \in \mathbb{Z}$.

page 261 **EXERCISE 11A.3.3**

14 Using the result of the previous question, show that the fourth power of any odd integer is of the form $8k + 1$.

page 265 **TEXT – Theorem 5**

Proof should reference Theorem 4, not Theorem 1.

page 272 **EXAMPLE 13** question should read:

Solve $172x + 20y = 1000$ in: **a** $\mathbb{Z}$ **b** $\mathbb{Z}^+$

page 278 **TEXT** top line of the page should read:

- A final type are those of the form $2^{2^n}+1$; these are the Fermat primes. $n = 3, 5$

page 293 **TEXT – THEOREM (FERMAT'S LITTLE THEOREM)** last four lines of the proof should read:

Thus, $a(2a)(3a)(4a)......(p-1)a \equiv (1)(2)(3)(4)......(p-1) \pmod{p}$

$\therefore \quad a^{p-1}(p-1)! \equiv (p-1)! \pmod{p}$

Now since $p \nmid (p-1)!$, p being prime, we can cancel by $(p-1)!$

$\therefore \quad a^{p-1} \equiv 1 \pmod{p}$

page 297 **TEXT** 6th dot point should read:

- Graph **5** is both W_4 and K_4.

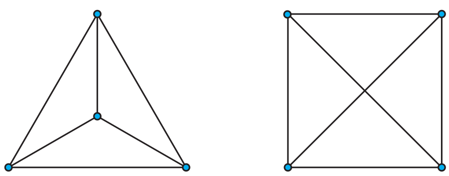
page 298 **TEXT** 6th definition should read:

Incident Edge/Vertex An edge which connects two adjacent vertices is said to be incident on each vertex.

page 302 **TEXT – GRAPH ISOMORPHISM** text under sub-section heading should read:

Isomorphism is an important concept in many areas of mathematics. You may have met it in other areas of the IB Higher Level Mathematics course, such as in Group Theory.

In **Section 11B.2**, we briefly introduced graph isomorphism when we compared the wheel graph W_4 with the complete graph K_4. We saw that these graphs have seemingly different representations on paper, as illustrated alongside, but they are in fact the same. Here they are again (and there are many other representations):



W_4 K_4

page 312 **TEXT** first blue box should read:

A connected graph is **Eulerian** if and only if all of its vertices are even.

A connected graph is traversable if and only if at most two of its vertices are odd.

page 352 **ANSWERS EXERCISE 8A**

4 $M \sim N(61, 11^2)$ and $C \sim N(48, 4^2)$
$U = M_1 + M_2 + M_3 + M_4 + C_1 + C_2 + C_3$
$U \sim N(388, 532)$
$P(U > 440) \approx 0.0121$ if unsafe
Assumption: The random variables M_1, M_2, M_3, M_4, C_1, C_2 and C_3 are independent.

page 360 **ANSWERS EXERCISE 8E.2**

4 b t-distribution with $s_{n-1} = \sqrt{\frac{389}{388}} \times \$0.25 \approx 0.2503$

page 362 **ANSWERS REVIEW SET 8A**

5 c ii F is now binomial

i.e., $F \sim \text{B}(12, \frac{1}{4})$ and

$$\begin{aligned} &\text{P(buy packet)} \\ &= 1 - \text{P(do not buy packet)} \\ &= 1 - \left\{\text{P}(F=2) + \text{P}(F=1) \times \tfrac{1}{4}\right\} \\ &= 1 - \left\{\binom{2}{2}\left(\tfrac{1}{4}\right)^2 + \binom{2}{1}\left(\tfrac{1}{4}\right)^1\left(\tfrac{3}{4}\right)^1 \times \tfrac{1}{4}\right\} \\ &\approx 0.844 \end{aligned}$$

page 374 **ANSWERS EXERCISE 10A.4**

3 c $\displaystyle\sum_{i=0}^{\infty} e^{-(i+1)^2} < \int_0^{\infty} e^{-x^2}\,dx < \sum_{i=0}^{\infty} e^{-i^2}$

4 c $\displaystyle\sum_{i=1}^{\infty} \frac{1}{(i+1)^2} < \int_1^{\infty} \frac{1}{x^2}\,dx < \sum_{i=1}^{\infty} \frac{1}{i^2}$

page 382 **ANSWERS EXERCISE 10D**

1

$$\begin{aligned} &\ln(1+x) \\ &= x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \ldots\ldots + (-1)^{n-1}\frac{x^n}{n} + R_n(x:0) \end{aligned}$$

Interval of convergence $]-1, 1]$.

$$\begin{aligned} R_n(x\!:0) &= f^{(n+1)}(c)\,\frac{(x-0)^{n+1}}{(n+1)!} \\ &= \frac{n!(-1)^n}{(1+c)^{n+1}}\,\frac{x^{n+1}}{(n+1)!} \\ &= \frac{(-1)^n}{(1+c)^{n+1}(n+1)}\,x^{n+1} \\ &\to 0 \quad \text{for} \quad |x| < 1, \quad -1 < c < 1 \end{aligned}$$

page 392 **ANSWERS EXERCISE 11A.3.3**

14 The square of an odd number has form $4p+1$, $p \in \mathbb{Z}$ (Question **13c**)

i.e., $a^2 = 4p+1$

$\Rightarrow$ $a^4 = (4p+1)^2 = 16p^2 + 8p + 1$

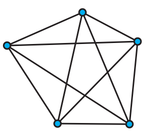
$\Rightarrow$ $a^4 = 8(2p^2 + p) + 1$

which is of the form $8k+1$, $k \in \mathbb{Z}$

page 398 **ANSWERS EXERCISE 11B.2**

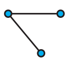
1 c iii 2, 2, 4, 4

3 a iv

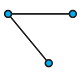


page 399 **ANSWERS EXERCISE 11B.2**

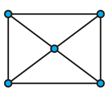
12 d



e



15 a



page 22 **CD-PRINTABLE CHAPTER "GEOMETRY" EXERCISE C**

10 Triangle ABC has altitudes [AP] and [BQ] where P lies on [BC] and Q lies on [AC]. H is the intersection of [AP] and [BQ].

Prove that AH.HP = BH.HQ.

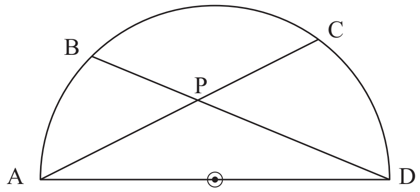
page 25 **CD-PRINTABLE CHAPTER "GEOMETRY" EXERCISE D**

8 [XAB] and [XC] are two intersecting straight line segments.

Given that BX = 6.4 m, AB = 5.5 m and XC = 2.4 m, prove that [CX] is a tangent to the circle through A, B and C.

page 26 **CD-PRINTABLE CHAPTER "GEOMETRY" EXERCISE D**

15



ABCD is a semi-circle with a diameter [AD].
P is the point of intersection of [AC] and [BD].
Prove that:

$$AP.AC + DP.DB = AD^2$$

page 30 **CD-PRINTABLE CHAPTER "GEOMETRY" EXERCISE E.3**

9 For $(x-2)^2+(y-2)^2=4$ and $x^2+y^2=4$:

11 **a** Find the equations of the radical axes of the circles $(x+2)^2+(y+4)^2=17$ and $(x-1)^2+(y-5)^2=5$. Do these circles intersect?

page 35 **CD-PRINTABLE CHAPTER "GEOMETRY" EXERCISE F**

(Remove question **12**)

page 38 **CD-PRINTABLE CHAPTER "GEOMETRY" EXERCISE G.1**

7 P is the midpoint of [BC] of triangle ABC. [PQ] is the bisector of angle APB and cuts AB at Q. [QR] is drawn parallel to [BC], meeting [AC] at R. Prove that angle QPR is a right angle.

page 41 **CD-PRINTABLE CHAPTER "GEOMETRY" EXERCISE G.2**

9 [AB] is a fixed diameter of a circle and F is a fixed point on [AB]. P and Q are points on the circle such that [PQ] is parallel to [AB] and is variable. Prove that $FP^2 + FQ^2$ is constant.

page 44 **CD-PRINTABLE CHAPTER "GEOMETRY" EXERCISE G.4**

4 **b** If the other diagonal has length y units, deduce that $y^2 = \dfrac{(ac+bd)(ad+bc)}{(ab+cd)}$

page 44 **CD-PRINTABLE CHAPTER "GEOMETRY" EXERCISE G.4**

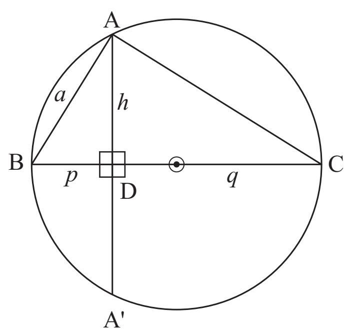
6 Similarly to **5**, use Ptolemy's theorem and the figure alongside to prove that $\sin(\alpha-\beta)=\sin\alpha\cos\beta-\cos\alpha\sin\beta$.

page 45 **CD-PRINTABLE CHAPTER "GEOMETRY" EXERCISE H.1**

2 Notice that Euclid's theorem (in a right angled triangle) was proved using similar triangles only. Use Euclid's theorem to prove that $a^2+b^2=(p+q)^2$, i.e., to prove Pythagoras' theorem.

page 46 **CD-PRINTABLE CHAPTER "GEOMETRY" EXERCISE H.1**

3



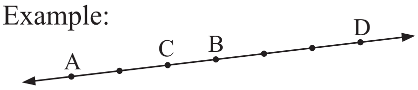
a [BC] is the diameter of a circle, centre O. A lies on the circle and A′ is the image of A under a reflection in the line [BC].

Use the 'intersecting chords' theorem to deduce that $h^2 = pq$.

b By drawing another circle on the diagram in **a**, prove that $a^2 = p(p+q)$ using the 'intersecting chords theorem - special case'.

page 47 CD-PRINTABLE CHAPTER "GEOMETRY" Harmonic Ratios replace text under diagram:

Example:



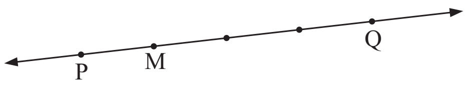
AC : CB = 2 : 1

and AD : DB = 6 : 3 or 2 : 1.

So, C and D divide [AB] harmonically and the harmonic ratio is 2 : 1.

page 47 CD-PRINTABLE CHAPTER "GEOMETRY" EXERCISE I

1



M divides [PQ] internally in the ratio 1 : 3. Locate N such that M and N divide:

a [PQ] harmonically

b [QP] harmonically. Illustrate.

page 51 CD-PRINTABLE CHAPTER "GEOMETRY" EXERCISE K

1 (remove part **b**)

2 P, Q and R lie on sides [AB], [BC] and [CA] of triangle ABC. If $AP = \frac{2}{3}AB$, $BQ = \frac{3}{4}BC$ and $CR = \frac{1}{7}CA$, prove that [AQ], [BR] and [CP] are concurrent.

page 58 CD-PRINTABLE CHAPTER "GEOMETRY" ANSWERS EXERCISE D

2 **d** $r = \sqrt{31}$ cm

4 **c** $OX = \sqrt{105}$ cm

page 58 CD-PRINTABLE CHAPTER "GEOMETRY" ANSWERS EXERCISE E.3

10 **a** (1, 3) and (3, −3) **b** $y = 6 - 3x$, $1 \leqslant x \leqslant 3$

c $y = 6 - 3x$

d $y = 6 - 3x$, the equation of the radical axis

11 **a** $x + 3y = 3$ No, they do not intersect.

page 58 CD-PRINTABLE CHAPTER "GEOMETRY" ANSWERS EXERCISE G.2

3 $\frac{\sqrt{208}}{3}$ cm, $\frac{10}{3}$ cm, $\frac{\sqrt{292}}{3}$ cm

page 58 CD-PRINTABLE CHAPTER "GEOMETRY" ANSWERS EXERCISE G.3

3 $\sqrt{105}$ cm

page 58 CD-PRINTABLE CHAPTER "GEOMETRY" ANSWERS EXERCISE H.1

1 **b** $\frac{144}{13}$ cm **c** $\frac{60}{13}$ cm

page 58 CD-PRINTABLE CHAPTER "GEOMETRY" ANSWERS EXERCISE H.2

2 **b** 16 : 33 **c** $\frac{98}{11}$ cm^2

page 58 CD-PRINTABLE CHAPTER "GEOMETRY" ANSWERS EXERCISE K

1 **b** 1 : 2

2 $\frac{AP}{PB} \times \frac{BQ}{QC} \times \frac{CR}{RA} = \frac{2}{1} \times \frac{3}{1} \times \frac{1}{6} = 1$ etc.

page 58 CD-PRINTABLE CHAPTER "GEOMETRY" ANSWERS EXERCISE L

4 1 : 2